Bellwether

Solving for X

A Primer on the Research, Practices, and Policies That Shape Math Education

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FEBRUARY 2026

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Introduction

Concerned by disheartening levels of student performance, education policymakers and practitioners are paying increased attention to math. Building on the momentum of evidence-based literacy reforms, there is a growing interest and urgency in identifying, developing, and implementing similar solutions to improve pre-K through Grade 12 math learning for all students.

Improving math learning will require unpacking what we know about the content students of all ages should learn, how best to teach that content, and what the most effective learning pathways are for students with diverse needs and aspirations. It will require leaders and practitioners to wrestle with the availability and quality of early learning; the cumulative nature of what students learn over their academic careers; the need to both address learning gaps and enable acceleration; and how capstone math courses affect students' postsecondary opportunities. State and local leaders will need to learn from the available research as well as the experiences of others implementing past and present policies to improve math performance.

This stocktaking report aims to build a shared, nuanced fact base on the current state of math research, practice, and policy.

"Mathematics is one of humanity's great achievements. By enhancing the capabilities of the human mind, mathematics has facilitated the development of science, technology, engineering, business, and government. Mathematics is also an intellectual achievement of great sophistication and beauty that epitomizes the power of deductive reasoning. For people to participate fully in society, they must know basic mathematics. Citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks."

—NATIONAL RESEARCH COUNCIL, 2001

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Executive Summary (1/3)

Math achievement in the United States is low, stagnant, and marked by stark inequalities.

- Although national data are limited, evidence suggests that the math performance of students entering kindergarten has either been flat or declining in recent years.
- Only 39% of fourth graders, 27% of eighth graders, and 21% of 12th graders are proficient in math, and many students score below even a basic level of foundational knowledge.
- Racial and socioeconomic achievement gaps in math are large and mostly unchanged since 2003.
- Math scores rose at similar rates for higher- and lower-performing students from 2003 to 2013, but since 2013, lower-performing students have fallen sharply while top-performing students have not.
- Low SAT math scores and high college remediation rates indicate that students are increasingly unprepared for college-level math.
- The U.S. ranks lower than most of its peer countries on international assessments of math.

The research on how students learn math is growing, but classroom practice remains fragmented.

- Research on effective math curriculum and instruction is less developed than in other domains like literacy with more focus on elementary grades rather than earlier and later years.
- There is broad consensus that learning math requires both conceptual understanding and procedural fluency, developed over time through cumulative instruction.
- Studies consistently find that students benefit from explicit instruction when learning new or complex skills, repeated and spaced practice to build fluency, and opportunities to apply knowledge through problem-solving and guided inquiry.
- Decades of debate over pedagogy, tracking, and course pathways — and inequitable access to best practices — have led to inconsistent instructional approaches within and across schools and states.

Executive Summary (2/3)

Effective math instruction follows a developmental progression across grade levels. However, not all students have access to instruction aligned with their needs and abilities.

- Math instruction progresses through a standard sequence of content, from foundations of number sense in pre-K to high school, when various math pathways lead to different capstone options with implications for students' postsecondary options.
- In early childhood and the lower elementary grades, guided play and concrete representations help children build foundational number and spatial understanding.
- Across all grade levels, instruction should integrate multiple approaches — including explicit teaching, guided inquiry-based learning, repetition and practice, feedback, and a gradual reduction in concrete and representational supports as students develop fluency and conceptual understanding.
- Efforts to target instruction to students' needs and abilities have included between- and within-class differentiation, as well as competency-based approaches.

Early childhood education (ECE) math is important but often neglected.

- Early math skills are among the strongest predictors of later academic success, college completion, earnings, and life outcomes.
- High-quality ECE math helps students learn concepts important for kindergarten readiness (e.g., counting, sorting, and pattern recognition), but it is not equally accessible to all young learners.
- ECE math is rarely a central focus of policymakers, and it is often subsumed under broader efforts to improve academic and program quality.

Executive Summary (3/3)

Efforts to improve math learning often face difficult implementation barriers.

- More than a century of math reform has produced uneven results, often due to misalignment between research, policy, curriculum, and classroom practice.
- Insufficient teacher preparation in mathematics, uneven adoption and use of high-quality instructional materials (HQIM) and aligned professional learning, and fragmented or unclear state guidance hamper teacher efficacy and student learning.
- A culture of math anxiety limits learning opportunities, growth mindsets, and performance.
- States often lack explicit, coherent, and evidence-based policies in mathematics.

In recent decades, math has been a primary focus of national and state reform efforts.

- Over the past 30 years, national math policy efforts have focused on standards, accountability, curriculum quality, acceleration, and course access.
- A number of states — such as Maryland, Ohio, and Alabama — have recently enacted policies designed to strengthen early math learning, identify and support struggling learners, improve student access to advanced math, and innovate around high school math pathways.

Improving math outcomes requires a coherent, cradle-to-career strategy that treats math learning as a cumulative progression.

- Successful reform balances conceptual understanding and procedural fluency, expands access to rigorous and relevant pathways, and supports educators with clear guidance, high-quality materials, and aligned professional learning.

To understand the debate around math instruction, it is valuable to know a few technical terms in mathematics. (1/2)

Key Terms
Ability Grouping: The practice of assessing students' learning needs and creating small groups for instruction, within the same classroom.
Automatic Enrollment Policies: The use of objective criteria to place students in the most advanced math classes for which they are eligible, without requiring them to opt in.
Cognitive Load: The amount of working memory used when the brain is processing information.
Cognitive Science: The study of how the brain learns.
Competency-Based Education: A form of instruction where students progress through content based on whether they have demonstrated mastery of a given concept rather than spending a particular amount of time in a class or course.
Conceptual Understanding: An educational goal in which students move beyond knowing how to do math to understanding why and how math works, including comprehension of mathematical concepts, operations, and relations.
Fixed Mindset: The belief that one's abilities, intelligence, and talents are static and cannot be significantly improved with effort.
Formative Assessment: An evaluation process that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes.
Growth Mindset: The belief that one's abilities, intelligence, and talents can be improved over time with effort.
High-Quality Instructional Materials (HQIM): Instructional resources (e.g., curricula) that are aligned to academic standards, are content rich with clear learning outcomes, reflect evidence-based practices, and provide a full suite of teacher and student materials.

To understand the debate around math instruction, it is valuable to know a few technical terms in mathematics. (2/2)

Key Terms
Inquiry-Based Instruction: An instructional approach in which students activate and apply learned knowledge and skills to problem-solving, sense-making, and reasoning.
Learning Progression: The sequencing of math concepts for learning development that build and develop student learning over time.
Math Fluency: The level at which students can use mathematical skills accurately, efficiently, and with reasonable speed, without having to struggle to recall or perform them.
Mathematical Reasoning: The ability of students to apply mathematical knowledge and skills to support logical thought, explanation, and justification.
Problem-Solving: The process by which students apply mathematical knowledge and reasoning to formulate, represent, and solve unfamiliar or complex challenges.
Procedural Fluency: The ability to apply math operations efficiently, flexibly, and accurately; transfer or modify operations to solve different problems and contexts; and recognize when one strategy is more appropriate to apply than another.
Scaffolding: The connection of new learning to prior knowledge, often by introducing new concepts through explicit instruction and then gradually moving students toward independent problem-solving.
Science of Reading: An interdisciplinary body of research about what works and matters most in teaching students to read that includes a set of fundamental pillars and skill-building progressions backed by causal studies grounded in cognitive science.
Tracking: The practice of assigning students to separate classrooms based on their needs and abilities.

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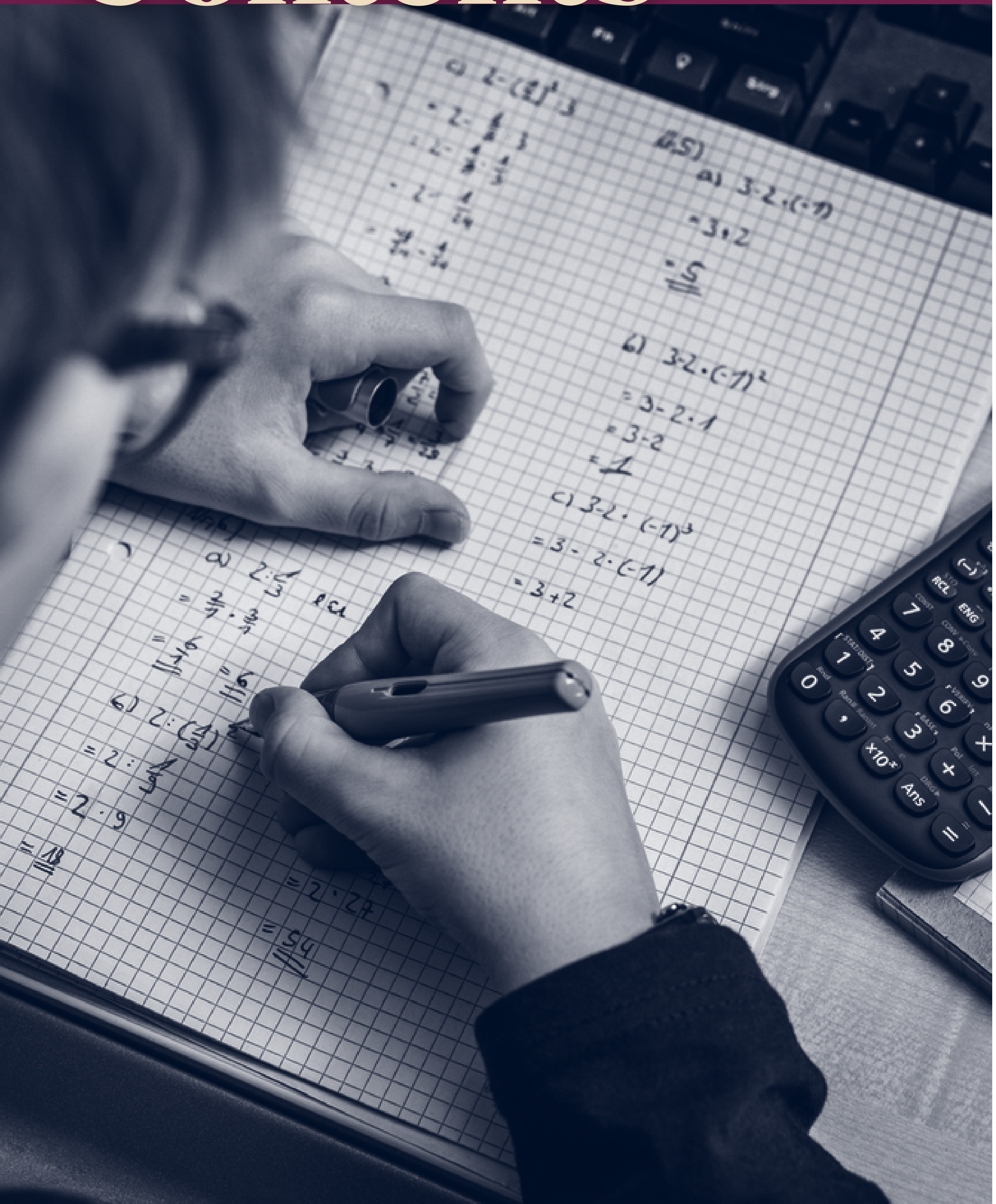


Students' acquisition of math learning is associated with long-term academic outcomes.

High School and Postsecondary Success

- **Early math skills at school entry can predict subsequent academic achievement even more powerfully than reading skills at school entry.** A meta-analysis found that school-entry math knowledge (e.g., numbers and ordinality) could predict later academic achievement in elementary and middle school, with effect sizes nearly twice as large as those for school-entry reading. Early math is also a more powerful predictor of later math achievement than early reading is of later reading achievement.
- **Early (K-5) math achievement is one of the strongest predictors of later educational success.** Children who experience persistent struggles in elementary math are 34 percentage points less likely to attend college and 13 percentage points less likely to finish high school compared to peers with stronger early math skills.
- **Students who score "Advanced" on the National Assessment of Educational Progress (NAEP)** eighth-grade math exam are about four times more likely to attend college and 30 times more likely to complete a four-year degree, compared with peers who scored "Below Basic."
- **Students who drop out of high school frequently cite their inability to pass algebra** as a major reason in their decision.
- **Taking more advanced math in high school strongly correlates with both college access and success.** Students who complete math beyond Algebra II (e.g., precalculus or calculus) have much higher odds of enrolling in and graduating from a four-year college.

Beyond academic outcomes, students' acquisition of math learning is also associated with socioeconomic outcomes.

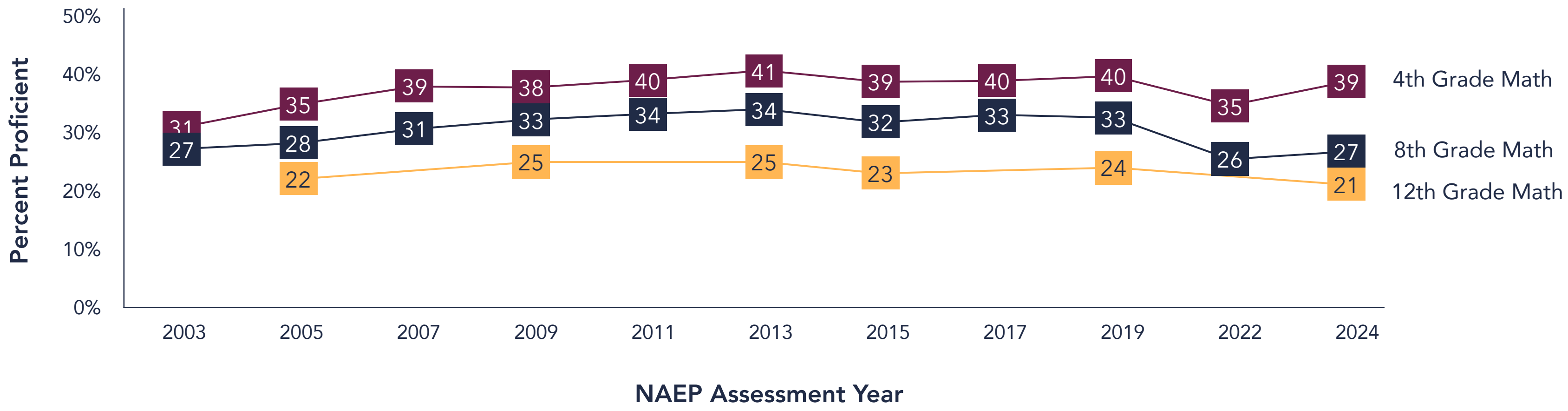
Socioeconomic Outcomes

- **Increases in early math achievement among children ages 5 to 8 are associated with larger increases in adult earnings** than comparable increases in reading achievement, health, or social relationships.
- **Improvements in math skills yield significant lifetime benefits.** A one-standard-deviation increase in eighth-grade math scores is linked to about 8% higher adult income and lower rates of teen parenthood, incarceration, and arrest even when controlling for an individual's race, age, geography, and parents' educational attainment.
- **Access to rigorous math coursework can close opportunity gaps.** One analysis of states' course requirements in mathematics found that stronger state course requirements in mathematics substantially increase Black students' math course-taking and boost their later earnings by roughly 10%, reducing Black-white gaps in coursework and future earnings.
- A longitudinal analysis of U.S. high school transcripts found that access to advanced math coursework (beyond Algebra II) has the potential to **reduce racial and socioeconomic disparities in bachelor's degree attainment by more than one-quarter**.

The academic and socioeconomic outcomes associated with math learning are especially significant in light of stalling math performance trends in the United States.

Math proficiency rates have declined since 2013, and recovery since the COVID-19 pandemic is incomplete.

NAEP Math Proficiency Rates Over Time in U.S. Public Schools, 2003-2024

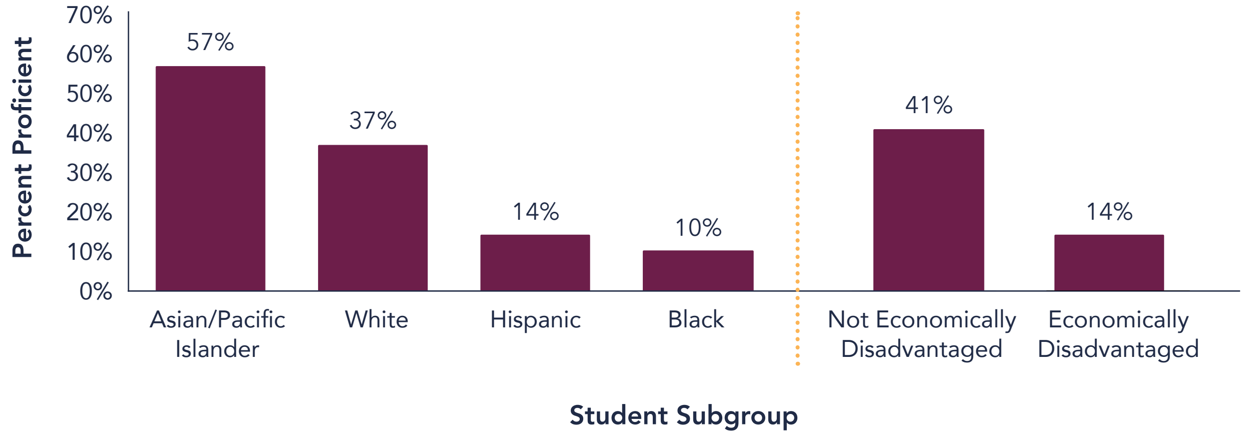


Although fourth-grade math scores have increased 8 percentage points since 2003, eighth-grade and 12th-grade scores are stagnant. A large majority of U.S. students today do not meet a minimum level of proficiency, including 61% of fourth graders and 73% of eighth graders.

Beyond the proficiency rates, 24% of fourth graders and 41% of eighth graders scored "below basic" on the 2024 NAEP exam, meaning that a student's performance falls below the minimum level needed to demonstrate even partial mastery of fundamental math knowledge and skills. For example, an eighth grader who scores below basic is not likely to be able to "find a missing angle in a triangle given two angles," "identify quadrilaterals," or "use a coordinate plane to identify and plot coordinate points precisely." Less than 10% of fourth graders and eighth graders scored "advanced," which indicates that they exhibit "mastery" of the competencies measured on the exam.

Stark inequalities in math performance exist across racial, socioeconomic, and achievement-level lines.

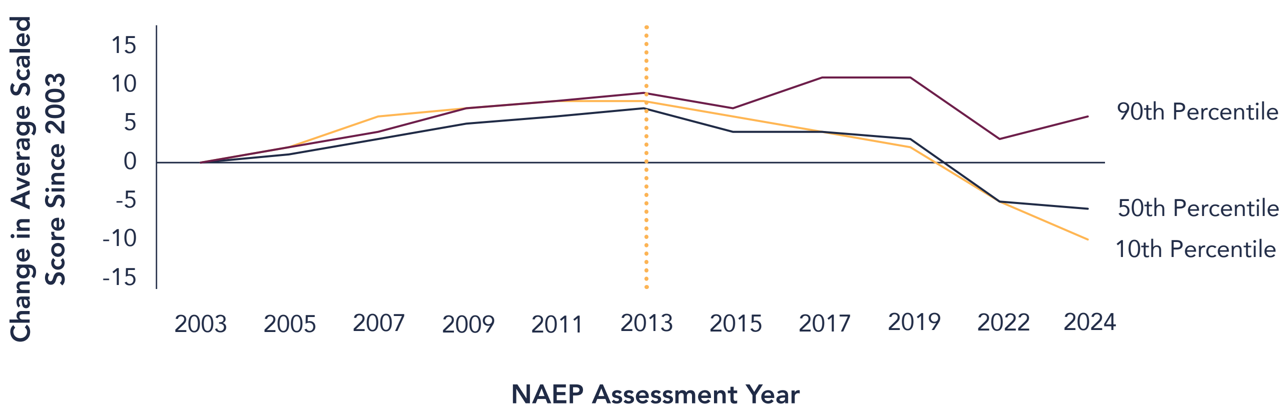
NAEP Eighth-Grade Math Proficiency Rates, by Race and Student Socioeconomic Status (SES), 2024



For all racial and SES subgroups, eighth-grade math performance increased between 2003 and 2013 and declined between 2013 and 2024. Racial and SES achievement gaps are nearly identical today to what they were in 2003.

NAEP Eighth-Grade Math Scaled Scores Change Over Time, by Achievement Level, 2003-2024

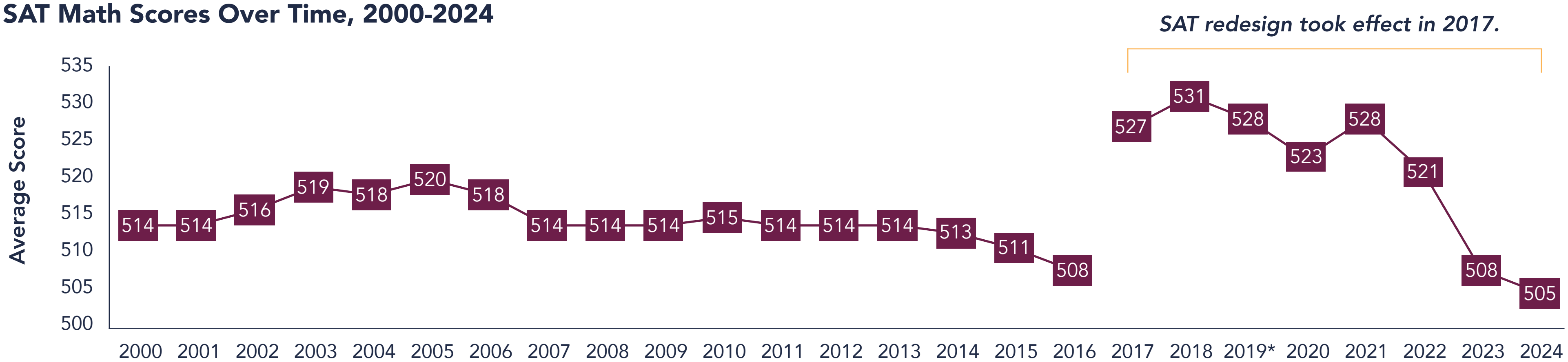
Scaled Score Point Change



Between 2003 and 2013, math scores increased at about the same rate for top-performing, average-performing, and low-performing students. Since 2013, there has been a sharp divergence in performance, with low-performing students dropping sharply compared with top-performing students.

Students are increasingly unprepared for postsecondary-level math.

SAT Math Scores Over Time, 2000-2024



**SAT participation peaked in 2019 at 2.22 million test-takers before dropping during the pandemic and the movement of many colleges and universities to "test-optional." In 2024, just under 2 million students took the SAT.*

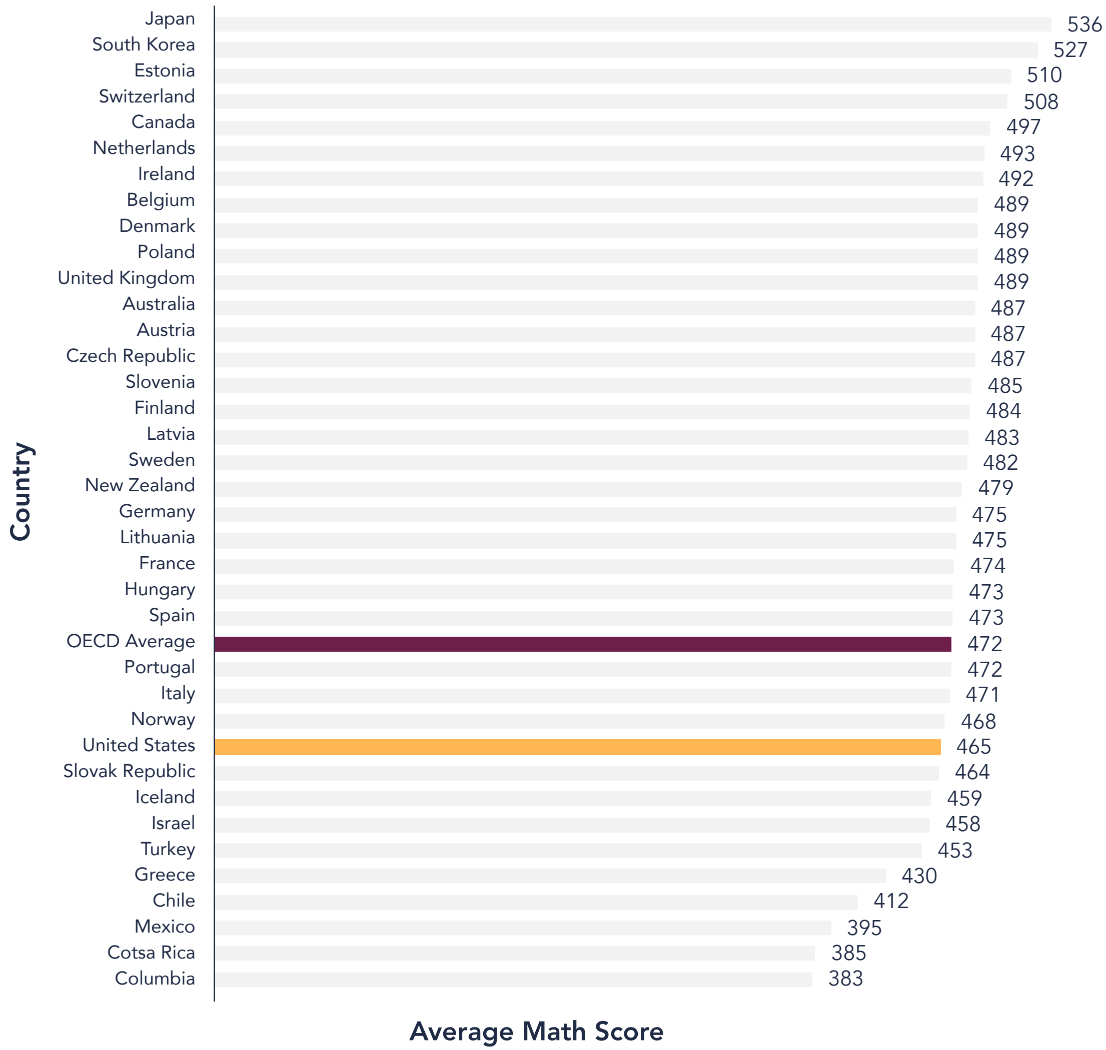
Despite a 2017 redesign that saw a jump in scores, SAT scores in 2024 are below 2000 levels. In 2024, just 41% of students who took the SAT "met the benchmark" in the SAT math exam — the level at which students can expect to have a 75% chance of earning at least a C grade in a first-semester, credit-bearing college-level math course.

College remediation rates are high. Forty percent of students in public two-year institutions and 25% of students in public four-year institutions reported taking a remedial course in 2019-2020 (the most recent year for which federal data is available). Of those students, approximately 64% reported taking at least one remedial math course, indicating that math is a major focus of remediation. A 2025 analysis of coursework at the University of California, San Diego (UCSD) found that nearly 900 first-year UCSD students in fall 2025 (approximately 12% of students) took a remedial math course, including more than 600 students who took a course that retaught elementary or middle school math. The number of students in remedial math has increased nearly 30-fold since fall 2020.

Notes*: These figures may even underestimate the problem as students sometimes do not know they are enrolled in a remedial course. In 2020, the University of California system stopped considering SAT scores in admissions. UCSD has argued that SAT math scores tend to be more predictive of college math placement than high school grades and that changes in admissions might be at least partially driving the increase in first-year student remediation.*

The U.S. ranks below many of its international peers in mathematics performance.

Programme for International Student Assessment (PISA), Math Scores, 2022



PISA is administered to 15-year-olds in participating countries internationally and assesses students' mathematical literacy and conceptual understanding. In 2022, the U.S. average math score was lower than the average math score in 27 of 36 peer Organisation for Economic Co-operation and Development (OECD) countries.

Since 2003, the U.S. has consistently performed below the OECD average, with scores declining over this period and reaching their lowest level ever in 2022. The U.S. has a higher percentage of students (34%) failing to reach basic proficiency on PISA compared with the OECD average (31%). The U.S. has a smaller percentage of students scoring advanced (7% versus 9%).

Despite concerning PISA scores, however, data that take into consideration countries' population size can paint a more positive picture of performance: The U.S. still ranks among the top countries in the raw number of high-performing math students.

Math proficiency is becoming increasingly critical as the U.S. and global economy evolves.

The U.S. economy's high-growth sectors demand a solid math foundation. Approximately one in four U.S. job postings asks for data science skills, and math-intensive jobs such as data scientists, operations research analysts, and actuaries are among the fastest-growing jobs in the U.S.

Even non-STEM careers such as manufacturing, skilled trades, and agriculture increasingly require mathematical knowledge and quantitative reasoning as they entail managing precision technologies, supply chain logistics, marketing research, predictive modeling, and resource sustainability. **Cross-national analyses of math performance indicate that countries excelling in math tend to have more innovation, stronger economic growth, and lower levels of economic inequality.**

Math achievement correlates more strongly with national innovation outputs (e.g., patent rates and new company formation rates) than achievement in reading or science.

Economists estimate that even a modest 25-point gain on a country's PISA math score can boost long-run gross domestic product by about 7%.

Countries with higher math performance tend to have lower wage inequality and poverty rates.

Concerns about math are creating momentum for math reform — and for a review of the evidence on what works.

A growing number of states are working to craft new math policies, and recent reports such as the Center on Reinventing Public Education's *State of the American Student*, the National Council on Teacher Quality's (NCTQ's) *States of the States: Five Policy Levers to Improve Math Instruction*, and 50CAN's *Mathways* illustrate a growing national interest in taking on this challenge.

As researchers, policymakers, and practitioners recognize, transforming math teaching and learning promises to be a complex task with deep historical roots tracing back decades. Meeting this challenge requires bringing together the best available evidence to build a shared and nuanced fact base about the current state of play in math research, instruction, and policy.

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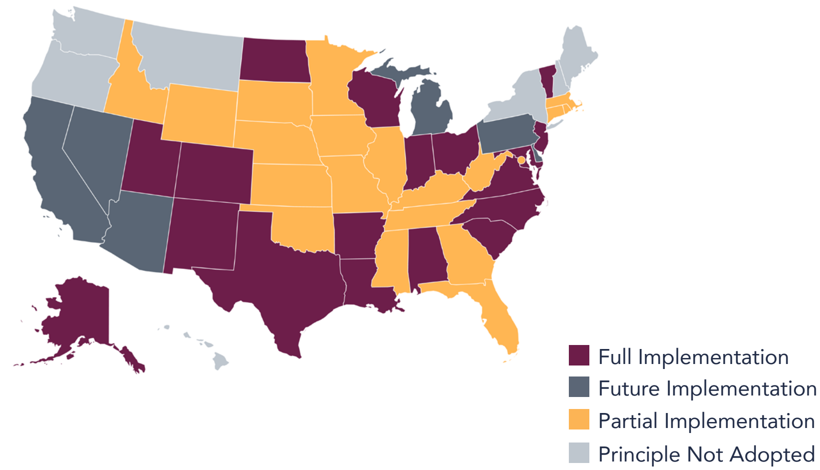
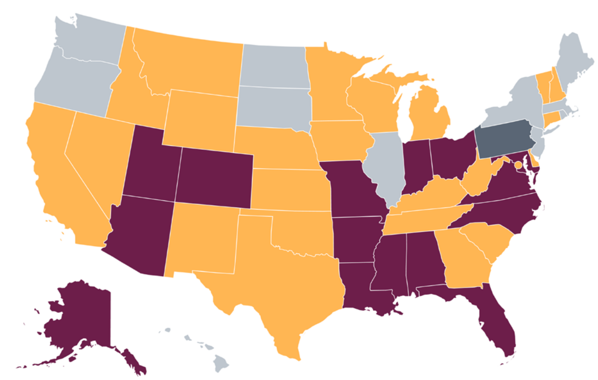
The Science of Reading movement has led to significant changes to literacy curricula, instruction, and policy.

The Science of Reading is an interdisciplinary body of research about what works and matters most in teaching students to read. It focuses on a set of fundamental pillars and skill-building progressions backed by causal studies grounded in cognitive science. Over the past several years, the Science of Reading movement has had many supporters across the political spectrum, including state and local policymakers, journalists, researchers, nonprofits, parent groups, and advocacy organizations. The momentum around the Science of Reading has spurred widespread changes in literacy policy and practice.

- Nearly every state has passed legislation related to implementing the Science of Reading since 2019. The breadth of those policies varies, and an existing statute does not necessarily cover critical domains necessary for quality implementation (e.g., support for teachers).

- The Science of Reading has been credited with driving dramatic improvements in literacy performance in states such as Mississippi, Louisiana, and Tennessee. Major school systems, such as New York City, have been overhauling curriculum and instruction to align with the Science of Reading.





 Source: *ExcelinEd, Literacy Map, "Science of Reading Training" and "Evidence-Based Interventions."*

Momentum around the Science of Reading has raised the possibility of creating similar momentum for math ...

... but the debates over math are fundamentally different from the debates in literacy.

Unlike in literacy, discussions about mathematics—what students should learn, how, and when—remain fragmented and, at times, contentious. Although literacy was a highly contentious field decades ago, today there is greater consensus around effective instructional practices.

In mathematics, consensus can be hard to find in policy and practice, particularly at the middle- and high-school levels. State-level math policies often lack coherence and grounding in high-quality evidence, if they exist at all. A few bright spots for math reform and student performance exist, but the lessons from those communities have not been widely shared or led to transformation across the field.

At the school level, higher-quality math curricula and instructional materials are being integrated into schools and classrooms, but full implementation and substantial improvements in teaching and learning remain elusive. These challenges are compounded by the fact that families, students, and even teachers can often doubt their own abilities to master mathematics.

Meanwhile, scientific research on how students build mathematical knowledge—particularly in cognitive science, human development, and instructional effectiveness—has matured in recent years, but engagement with this research is not always evident in policy and practice.

There are significant obstacles to establishing a "Science of Math" and replicating the momentum of the Science of Reading.

1. A less unified research base about what drives effective math learning.
2. Recurring "math wars" about how math should be taught.
3. Fixed mindsets about math aptitude and a culture of math anxiety.
4. The cumulative nature of math learning, which can continue indefinitely.
5. The absence of an organized political and social movement.

1 The research base about what drives effective math learning is smaller and less unified than it is for reading.

The research base in math is smaller and less oriented toward causal research. The research base includes many qualitative and mixed methods studies that probe outcomes beyond student achievement, including attitudes, self-efficacy, and conceptual understanding, which are harder to measure. There are notable gaps in research at the ECE and high school levels.

Math research does not lend itself to a clear and unified theory about how kids learn math, in part because math is a collection of many different concepts and skills. Although promising findings exist (e.g., on effective strategies for teaching elementary addition), they do not coalesce into a single recipe for instruction across all math concepts and skills.

This has led to major discussions among researchers and thought leaders about whether a single evidence-based Science of Math is possible to achieve.

"What has allowed research on learning to read to capture the imagination of journalists, parents, teachers, etc., is a single, unified story that we can tell about how children learn to read. … The situation in math is just very different. There isn't that kind of unity, which means we have to rebuild a "Science of _____" for each mathematical topic. And we haven't gotten to many of them yet." —MICHAEL PERSHAN, MATH EDUCATOR AND WRITER

2 Unresolved debates lead to recurring “math wars” over what effective math instruction should look like.

Math education has been marked for decades by debates over how best to teach the subject — namely, how much emphasis should be placed on things like procedural fluency, conceptual understanding, teacher-directed instruction, and inquiry-based instruction.

In math, consensus has long existed that instruction should balance procedural fluency, explicit instruction, and conceptual understanding, but disagreements persist over where the appropriate balance point lies.

Efforts to introduce more conceptually oriented math lessons often receive pushback from parents and teachers accustomed to traditional methods, while efforts to return to procedural lessons and fundamentals often spark fear of abandoning deeper understanding. Fears of instruction swinging too far from one extreme to the other remain prevalent.

Since 2020, an organized group of mostly special education educators and researchers has attempted to establish a “Science of Math” movement advocating for explicit, systemic, teacher-directed instruction for all students.

Several mathematics education associations challenged this movement in 2025, declaring it “self-branded” and “not the primarily endorsed evidence-based approach” while advocating for greater conceptual understanding, problem-based learning, and student engagement alongside procedural fluency.

> *“The problem with the term ‘Science of Math’ is that you’re trying to simplify something that is not simplifiable. … If [the Science of Math] were used as a communications vehicle [like in reading], then nuance is going to be left behind and instruction’s going to swing to one extreme.”*
>
> —JOSHUA PARRISH, DIRECTOR OF COMMUNICATIONS AND ENGAGEMENT, COLLABORATIVE FOR STUDENT SUCCESS

3 Although all students are expected to become strong readers, the same belief is not held for math.

There is a cultural myth that being good at math might be an inborn trait — that some people are "math people" and others are not. This fixed mindset blunts the urgency of math reform and normalizes low expectations and performance for students. Math generates much more anxiety and aversion among students and adults than reading. Math identities start to form as early as preschool and are largely set by the end of elementary school.

64%

of Americans struggle with math anxiety.

25%

of teachers feel anxious doing math.

37%

of American adults report exclusively negative feelings toward math.*

> *"Illiteracy in math is acceptable the way illiteracy in reading and writing is unacceptable. Failure is tolerated in math but not in English."* —BOB MOSES, FORMER CIVIL RIGHTS ACTIVIST AND EDUCATOR

Note: **Women (43%), Hispanics (44%), and adults without a bachelor's degree (42%) have the highest rates of exclusively negative emotions toward math.*

4 Math skills continuously accumulate and can grow indefinitely.

Students never truly finish learning to do math. Math is a “ruthlessly cumulative” subject in which skills continuously and recursively build with no clear endpoint. In literacy, there is a more finite goal for students: to become proficient readers who can comprehend and learn from text. In other words, they stop learning to read and begin reading to learn.

The progression and sequence of math learning extends from early childhood through college and beyond, branching into different tracks along the way that have significant implications for students’ career possibilities. In literacy, skill development is front-loaded in the early years; it enables learning in later years of schooling and is necessary for success in any future career.

The sequential, more hierarchical, and career-connected structure of math learning feeds debates about tracking and who should have access to what courses — debates that are not present or relevant in literacy development for early learners.

> *“There are so many different outcomes that people are trying to achieve, and so many different understandings about what it means to do math. It makes it very hard to figure out one approach for learning math even in the context of common expectations.”*
> —EBONEY MCKINNEY, PRESIDENT, ASSOCIATION OF STATE SUPERVISORS OF MATHEMATICS; DIRECTOR OF MATHEMATICS AND EDUCATIONAL TECHNOLOGY, ARIZONA DEPARTMENT OF EDUCATION

5 Efforts to build a science of math lack an organized political coalition and social movement of support.

The differences from the Science of Reading may make an analogous Science of Math less likely, but progress on math reform nonetheless requires understanding both the research behind math instruction and learning and the considerations that continue to influence the path forward.

Science of Reading Movement	State of Math Reform
News media champions, such as journalist Emily Hanford and her "Sold a Story" podcast, galvanized public outrage about ineffective reading instruction.	Math has had no comparable breakthrough in popular media.
Reading advocates unified around a clear, accessible message and framework: "the Science of Reading" and its five pillars.	No Science of Math framework has achieved this level of simplicity, clarity, and coherence — nor is a similar framework in math necessarily possible.
Organized parent activism, especially from families of students with dyslexia, helped create pressure for reading reforms.	Math does not have a similar grassroots parent movement. Parents of children with dyscalculia (a math learning disability) are less organized and visible, although conversations about the issue are growing more frequent.
The Science of Reading movement has had high-visibility political champions across party lines, many of them governors, chief state school officers, and state legislators.	Few math policy champions exist, and the issue does not offer the same easy political wins as the Science of Reading.

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Effective practice in math teaching and learning draws from evidence associated with four major research domains.

Effective math practice draws from four major research domains, each with well-documented findings about how students learn mathematics effectively.

Cognitive Science	Evidence about how the brain learns.
Learning Progressions	Evidence about the appropriate sequencing of math concepts for learning development.
Mathematical Reasoning and Problem Solving	Evidence about how students develop and apply mathematical knowledge and skills to achieve math fluency.
Mindset	Evidence about how to foster a growth mindset in mathematics to promote motivation and learning.

Cognitive science research provides insights into how the human brain processes and retains information. (1/2)

Two mental processes are critical to learning: memory and cognitive load. Memory refers to the way the human mind receives and retains information. It includes both working memory and long-term memory.

Working memory is where information is processed and initial learning takes place.

Encoding is how information from working memory is transferred to long-term memory. Information that is not encoded is forgotten.

Retrieval is how information from long-term memory is brought back into working memory. Repeated practice of retrieval can strengthen long-term memory.

Long-term memory is where previously learned information is stored. Long-term memory has virtually unlimited capacity.

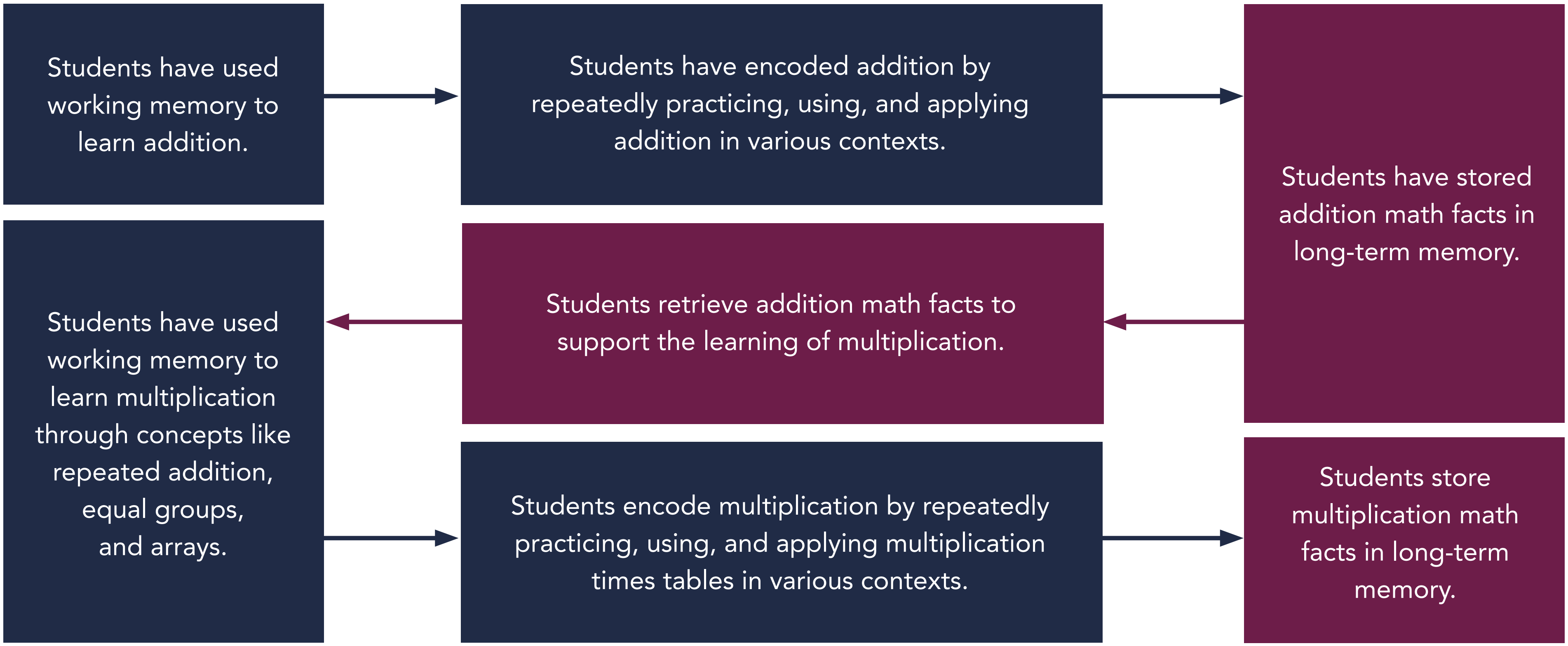
Cognitive load is the amount of working memory being used when the brain is asked to process information. Long-term memory is virtually unlimited, but working memory is limited. As a result, learning progresses best when cognitive load is well managed.

Small amounts of information are processed in working memory and then encoded to long-term memory.

Information in long-term memory is retrieved to short-term memory, as needed, to build on existing knowledge.

Cognitive science research provides insights into how the human brain processes and retains information. (2/2)

For example, when students learn multiplication, several cognitive processes take place:



When multiplication math facts are stored in long-term memory, students' ability to retrieve those facts quickly and easily then reduces the cognitive load they experience when learning new math skills and concepts.

***Note**: This graphic is a simplified model of how the brain works.*

Effective math instruction uses four mutually reinforcing strategies to support encoding and retrieval.

Scaffolding involves connecting new learning to prior knowledge, introducing new concepts through explicit instruction, then gradually moving students toward independent problem-solving. It allows students to limit cognitive load during initial learning, building fluency gradually and supporting integration of new concepts into existing mental frameworks.

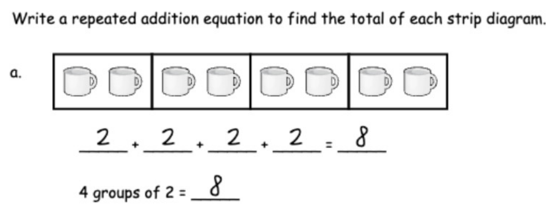
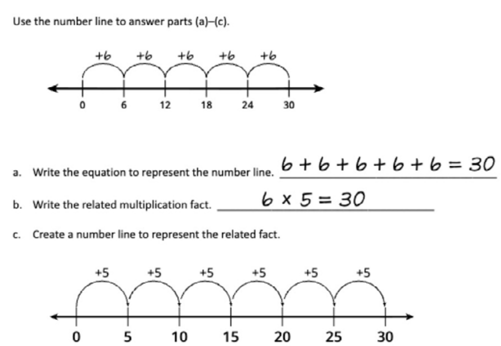
Retrieval practice, or problem sets of newly learned skills, provides students the opportunity to build fluency by moving skills from working to long-term memory. This "encoding" process frees up working memory, reducing the cognitive load needed to master the next skill. Retrieval practice, also referred to as delayed retesting, works best when it is spaced out over time.

Cycles of feedback and reflection can support the development of independent problem-solving skills while limiting cognitive load. Instructor feedback can be used to identify specific errors in student reasoning before they become a habit. Reflection on those errors allows students to correct their reasoning, increasing fluency and deepening conceptual understanding.

Desirable difficulty, often referred to as "productive struggle" or the "zone of proximal development," refers to achieving a "just-right" level of challenge for students by slowly increasing the difficulty of tasks as students master the initial skill. Often, the appropriate level of productive struggle is easier to foster at the individual student level than at the classroom level.

Learning progressions use cognitive science to build evidence-based road maps for teaching and learning.

Students often take individual learning paths to progress from foundational to more sophisticated concepts, building on existing knowledge and skills along the way. Learning progressions describe the process by which student learning evolves over time. Math standards are designed with intended learning progressions across the pre-K through Grade 12 continuum.

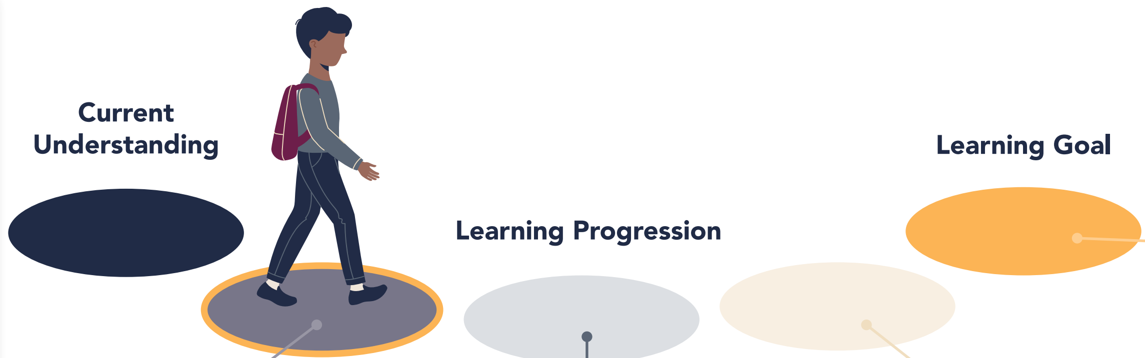
	Standard	Student Task	
Grade 2	Model, create, and describe contextual multiplication in which equivalent sets of concrete objects are joined		**In this example, students use an array to understand the concept that multiplication is repeated addition.**
Grade 3	Represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting		**Now, students are creating number lines and using equal jumps to solve multiplication problems.**

 Sources: *Bluebonnet Learning K-5 Math, Edition 1, "Grade 2: Module 6" and "Grade 3: Module 3."*

Learning progressions use theory and observation to move students from their current understanding to a learning goal.

Because content standards are too all-encompassing for daily lesson planning, **learning progressions can be thought of as building blocks** or the intermediate steps a student needs to take to achieve a standard. It is important to note that although learning progressions describe a common path that students can take toward a learning goal, each student is different and may require different steps.

Standard: Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 x 7.



Recognize the structure of repeated addition and understand that it can be expressed as the number of times a quantity repeats

Practice repeated addition of objects arranged in rectangular arrays with progressively more rows and columns (more than 5 rows and 5 columns)

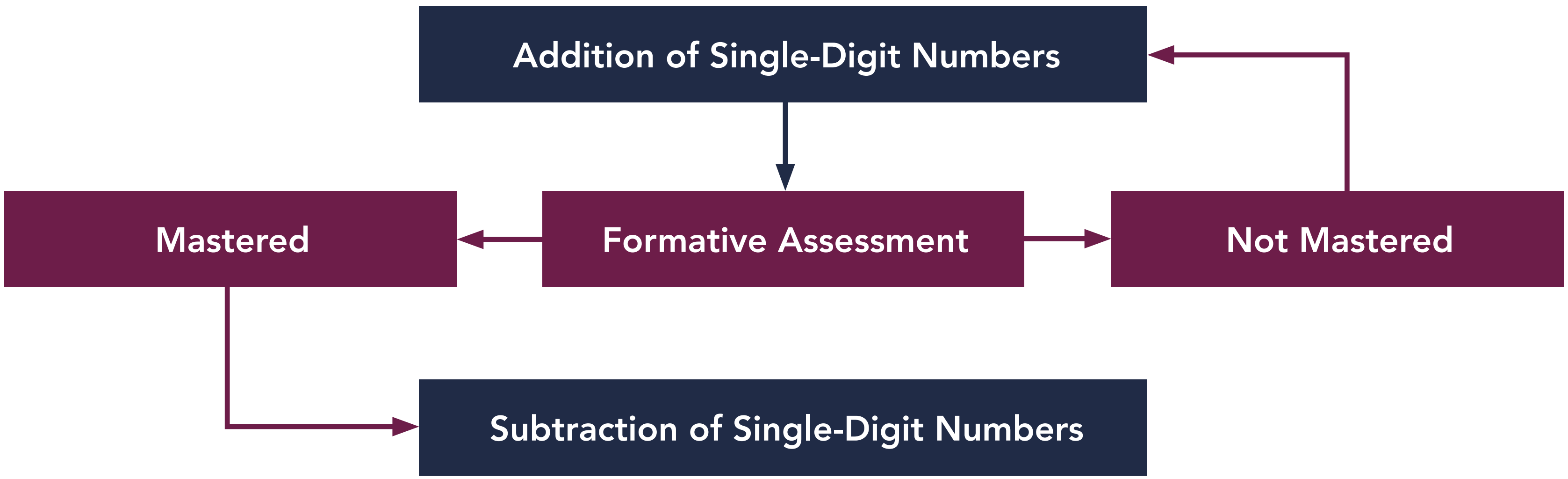
Move between symbolic (2 + 2 + 2 + 2) and concrete (four groups of 2 objects) representations of the same repeated addition sentence

Describe repeated addition like 2 + 2 + 2 + 2 as "the number 2, added four times," and then "four times 2"

 Source: *Common Core State Standards (CCSS), Math Content 3. OA.A.1.*

Learning progressions and formative assessments help maintain instruction at the level of desirable difficulty.

Learning progressions reflect how students' understanding typically develops. For example, the learning of addition typically precedes the learning of subtraction.



A formative assessment helps a teacher understand what a student can do independently, what a student can do with guidance, and what a student is not yet able to do. Based on that information, the teacher can differentiate instruction — revisiting past content or teaching new content. This helps teachers keep students at a level of desirable difficulty.

Conceptual understanding of math underpins procedural fluency, mathematical reasoning, and problem-solving.

Conceptual understanding is when students move beyond knowing how to do the math to understanding why the math works. For example, 2 + 2 means putting two units together with another two units. 4 > 2 because putting two units together with two other units creates a value that is greater than two units alone.

Procedural fluency involves executing mathematical operations accurately, efficiently, and flexibly. Students develop procedural fluency by knowing the procedures, identifying the right procedures for a familiar problem, and performing those procedures accurately and with low working memory.

Example: A student would recognize the algebraic expression "a + b = c" and be able to calculate the value of "a" when given the values of "b" and "c."

Mathematical reasoning involves making sense of mathematical ideas, as well as understanding how those ideas fit together into the larger frameworks of mathematical systems. Students develop math reasoning skills as they figure out how to effectively explain the steps they took to solve the problem.

Example: A student who knew how to determine the value of "a" would use mathematical reasoning to explain the process used to get to the answer.

Problem-solving involves applying mathematical knowledge and reasoning to unfamiliar or complex situations where the solution path is not obvious. Students develop problem-solving skills as they figure out how to apply previously learned math knowledge and skills to new situations.

Example: A student who understands the expression "a + b = c" would figure out how to apply that expression correctly to a word problem.

The central role of desirable difficulty in mathematical proficiency underscores the importance of student mindsets.

Mindset is how humans think about their own abilities. There are two main types of mindsets: fixed (ability is inherent and static) and growth (ability can be improved through effort and perseverance). The level of desirable difficulty in instruction, learning progressions, and the development of conceptual understanding, procedural fluency, mathematical reasoning, and problem-solving is essential for maintaining a growth mindset. When students develop a fixed mindset, it gets in the way of the development of mathematical proficiency. The same applies to teachers.

Fixed Math Mindset

"I am not a math person."

Students and teachers with a fixed math mindset believe math ability is inherent: Someone either has it, or they do not. Students with fixed math mindsets tend to give up after initial setbacks or errors; teachers with fixed math mindsets tend to validate struggling students' conception that they have lower math fluency.

VS.

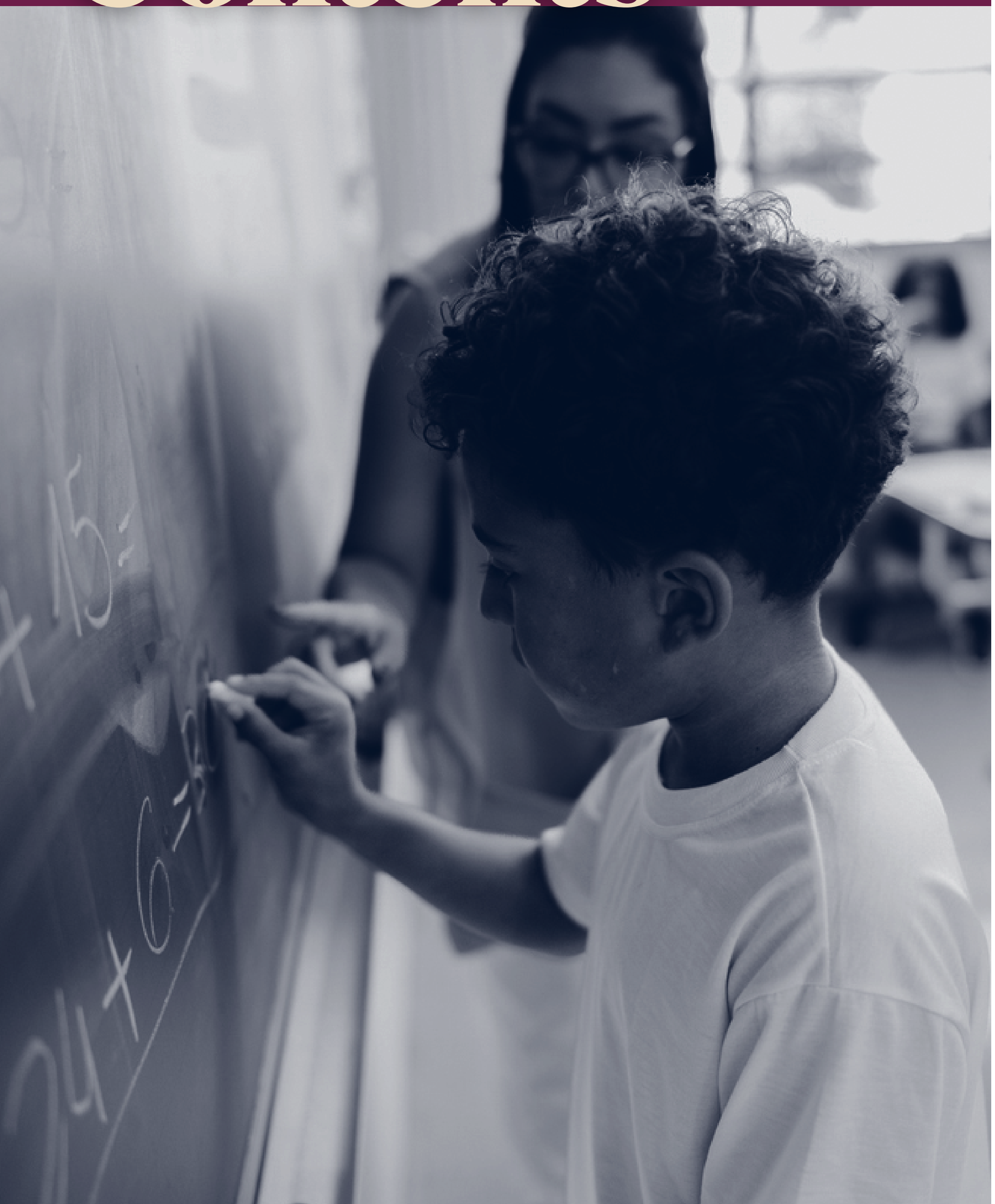
Growth Math Mindset

"I can build my math skills through persistent practice."

Students and teachers with a growth math mindset believe that math ability develops with attention, learning, and practice. Students with growth math mindsets tend to persist after making errors; teachers with growth math mindsets tend to encourage students to see math ability as something to be cultivated over time.

Absent effective instruction and opportunities to learn, a growth mindset by itself will not lead students to develop proficiency in math. However, it is a critical condition for learning. When students are confident in their abilities, they are more likely to engage and demonstrate improved achievement on math standardized tests and have higher enrollment rates in advanced math courses. Moreover, when students believe they can become proficient in math, they are more motivated to engage in the persistent practice that results in math fluency.

Contents



Common considerations in math education center around three themes.

Theme 1

How Math Is Taught

Theme 2

What Math Skills, Content, and Courses Are Taught

Theme 3

Who Receives What Instruction

Theme 1

How Math Is Taught

There are five common approaches to math instruction, with some variation based on student age and grade level.

Early Childhood (Ages 3-5)	Lower Elementary (Grades K-2)	Upper Elementary (Grades 3-5)	Middle School (Grades 6-8)	High School (Grades 9-12)

GUIDED PLAY

Students engage in adult- and student-initiated play that is connected to learning goals.

INQUIRY-BASED LEARNING

Students activate and apply learned knowledge and skills to problem-solving, sense-making, and reasoning.

EXPLICIT INSTRUCTION

Students receive explicit, step-by-step teaching of math concepts in small, coherent chunks building on prior skills.

REPETITION AND PRACTICE

Students repeatedly retrieve knowledge and apply foundational arithmetic skills through spaced, cumulative, and sometimes timed activities.

CONCRETE-REPRESENTATIONAL-ABSTRACT (CRA)

Students rely on hands-on experiences when learning a new concept, moving to drawings and eventually to symbolic notation as they become more proficient.

In early childhood math, free play, guided play, and explicit instruction are debated approaches.

	Free Play	Guided Play	Explicit Instruction
Definition	Child-led play that is unstructured, spontaneous, and free of adult intervention.	A combination of child-led play and teacher-led instruction that typically involves exploring and manipulating concrete materials, active problem-solving, and reasoning. Play is in service of a math learning goal.	Teacher-led instruction focused on systematically building skills and teaching factual knowledge.
Rationale	Promoted as an opportunity for children to transfer, use, and develop everyday concepts naturally through imagination and pretend play, which leads to the development and construction of mathematical knowledge.	Promoted as a developmentally appropriate approach to learning that focuses on the whole child and socialization while also supporting the development of deep mathematical understanding.	Promoted as an approach to close achievement gaps for students who enter school without the requisite skills and knowledge (disproportionately children from low-income households).
Critique	Criticized as insufficient for teaching children math, leading to little or no durable learning taking place.	Without the right level of guidance, students can experience a cognitive overload, resulting in little to no long-term memory of skills or concepts learned.	Criticized as a "drill and kill" approach that relies on counting by rote and worksheets but fails to help children understand concepts they will need to build on in later math.

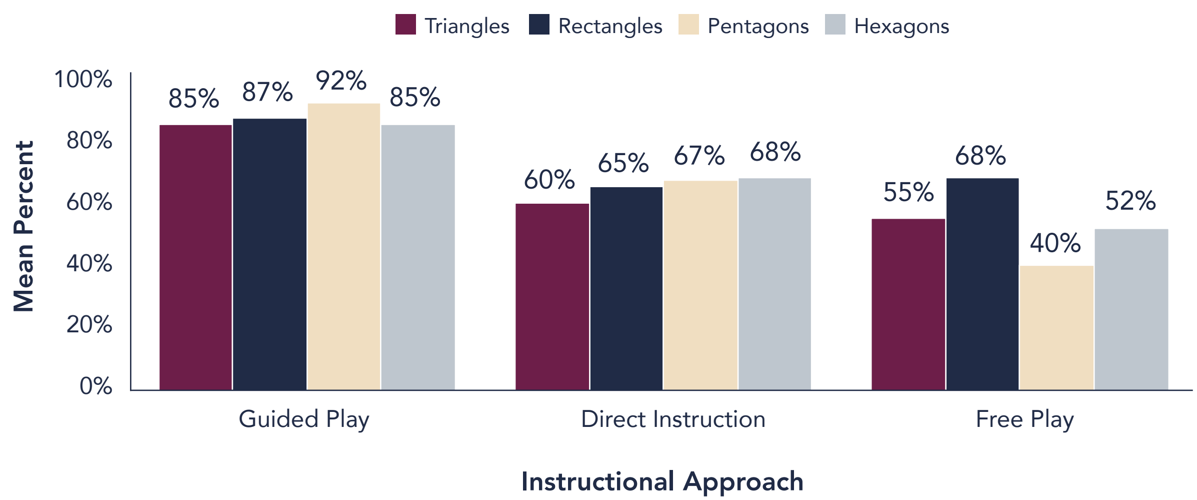
Guided play has emerged as a leading instructional approach for early childhood math …

Experts, including the National Council of Teachers of Mathematics, National Association for the Education of Young Children, and the National Academies of Science, Engineering, and Medicine, have converged around guided play as the leading instructional approach to math instruction. In fact, the two most widely used preschool curricula, The Creative Curriculum and HighScope, are grounded in it.

Research supports guided play as an instructional approach.

- A systematic review and meta-analysis of 39 studies found guided play had positive effects on early math skills, shape knowledge, and task switching.
- One study found that less affluent children eliminated differences in numerical estimation proficiencies after playing numerical board games. For example, low-income children increased their ability to place numbers in the correct order from 61% to 81% — nearly identical to their higher-income peers.
- A study of 4- and 5-year-olds found those who received guided play instruction correctly identified and sorted shapes based on their features at higher rates than those who received direct instruction and free play.

Mean Percent of Typical Shapes Correctly Sorted as Real, by Instructional Approach



… but the utility of guided play is only as good as its implementation.

Effective guided play instruction depends on the level of guidance students receive, teacher capacity, and whether it is used in conjunction with other instructional approaches.

- **The Level of Guidance**: Cognitive load theorists argue that any approach with reduced guidance can overload young children's very limited working memory, especially when content is new or complex. Guidance that is not specific can become inefficient and frustrating rather than productive.
- **Limits on Teacher Pedagogy**: Guided play is pedagogically demanding; without strong teacher content knowledge, scaffolding skill, and planning, it can result in vague goals, uneven support, and missed opportunities — with potential equity implications for students with disabilities or those facing academic difficulties.
- **Used Absent Other Approaches**: If guided play is used as the primary method, without enough explicit modeling and practice, students may not develop the foundational skills necessary for later math. Novice learners in particular benefit most from explicit guidance. And although free play is not ideal for specific learning goals, research does indicate that allowing children time to explore objects or pretend play can help them develop important math and spatial skills along with social, emotional, and intellectual development.

There are three approaches to guided play instruction, where students and teachers take turns initiating learning.

A study of early childhood educators identified three approaches teachers take to guided play instruction: extending, facilitating, and inviting. And because guided play is strategic and connected to learning goals, teachers plan with students prior to play and debrief with students after play.

Before Play	Guided Play			After Play
Planning Prior to any play, teachers help children plan and consider their intentions. Strategies include class discussions, voting, and entry tickets.	**Extending** Children initiate play. The teacher extends the play toward math goals by offering materials, commenting on play using specific math language, joining in play, and asking questions.	**Facilitating** The teacher helps children to coordinate and accomplish their goals in play. The teacher also co-constructs during guided play and co-directs inquiry through conversation about play and learning.	**Inviting** The teacher provokes or entices students by creating a playful context to target specific math goals. Children join and play within this adult-created context.	**Debriefing** After play activities, teachers gather students to recap what they did and what they learned. Strategies include sharing any documentation of play, celebrating learning, and completing exit tickets.
	Child-Led	**Mutually Led**	**Teacher-Led**	

"One danger of play-based instruction is that people think it is just play. They don't identify all of the cognitive, social, physical, linguistic domains that are happening during that play. That's essential, and it's also essential for kids who are playing to learn math, to know that they're learning math. To say, 'This is math time. We are going to go to these four play-based stations. You're going to do math in each one. You're going to tell me about the math you're doing, you're going to describe the math, you're going to show me.'" —CARRIE CUTLER, PROFESSOR, UNIVERSITY OF HOUSTON

Best instructional practices for ECE are not equally accessible to young learners.

Access to quality instruction, materials, and qualified teachers varies by ECE setting. In systems that deliver ECE through a mix of providers, public-school classrooms tend to provide more and higher-quality math instruction and have more qualified instructors than community-based organizations (CBOs) such as for-profit, parochial, or religious preschools and nonprofit organizations. Because CBOs and Head Start programs serve more children of color, dual language learners, and students from low-income households, ensuring parity in materials, coaching, and protected math time is essential to achieving comparable outcomes across settings.

Students from low-income households, students of color, and immigrant children receive fewer opportunities to experience play-based pedagogy. Children from low-income households receive more direct, whole group math instruction and have fewer opportunities to use manipulatives compared with their higher-income peers.

The amount of time early childhood students spend on math during the school day is insufficient, especially when compared with literacy. One study found that during 120-minute observations of pre-K and kindergarten classrooms, students spent an average of about 6 minutes and 30 minutes, respectively, on math compared with an average of 50 minutes on literacy-focused activities. Research shows that a math-focused curriculum can increase the amount of math instruction students receive.

Inquiry-based learning is intended to activate students' prior knowledge and experiences to work through a task.

Both guided play and guided inquiry are learner-active and goal-directed instructional approaches, with teacher scaffolds (questions, modeling, success criteria). However, as students get older, they work more with symbols, models, data, and proofs rather than manipulatives, blocks, and "centers."

There are active debates on the effectiveness of inquiry-based learning, and evidence is mixed depending on the amount of guidance students receive. Two meta-analyses of 164 studies comparing guided inquiry, unguided inquiry, and explicit instruction found that explicit instruction produces more favorable outcomes when compared with unguided inquiry, but guided inquiry produced more favorable outcomes than both.

Minimal or Unguided Inquiry	When presented with a problem, students are expected to discover or construct essential information on their own. Cognitive load theorists argue that the level of cognitive demand on a student's working memory required by minimal or unguided inquiry leaves little capacity to store any new information learned in a student's long-term memory. Other critics argue that "students from disadvantaged backgrounds … often lack the cultural capital to navigate discovery-based approaches" such as inquiry-based learning.
Guided Inquiry	With guided inquiry, on the other hand, students receive teacher supports, such as scaffolds, explanations, and worked examples. A meta-analysis of 72 studies on guided versus unguided inquiry math and science instruction found that guided inquiry had a significantly positive effect on learning activities, performance success, and learning outcomes.

Inquiry-based learning places higher-level cognitive demands on students.

Inquiry-based learning requires significant cognitive effort on the student's part. The student must analyze the task before them, access relevant knowledge and experiences, and leverage their understanding of mathematical concepts, processes, or relationships to solve a given problem. Take these examples of inquiry-based learning:

- Each promotes reasoning and problem-solving and requires students to access previous knowledge.
- **Example A** and **Example B** invite students to explore multiple approaches to solving the task, while **Example C** asks students to explore how changing values impact exponential functions.

Example A

You are trying to decide which of two smartphone plans would be a better value. Plan A charges a basic fee of $30 per month and 10 cents per text message. Plan B charges a basic fee of $50 per month and 5 cents per text message.

How many text messages would you need to send per month for Plan B to be the better option? Explain your decision.

Example B

There are 10 cars in the parking lot. Some of the cars are red and some of the cars are black. How many red cars and how many black cars could be in the parking lot?

Think of as many different combinations of cars as you can. Show your solutions in as many ways as you can with cubes, drawings, or words, and write an equation for each solution.

Example C

Using your graphing calculator, investigate the changes that occur in the graph of $y = a^x$ for different values of a, where a is any real number. Explain what happens in the following cases:

1. $a > 1$
2. $a = 1$
3. $0 < a < 1$
4. $a = 0$
5. $a < 0$

Cognitively guided instruction (CGI) is an evolving pedagogical approach supporting inquiry-based learning.

CGI is a student-centered framework to support teachers in planning math instruction based on how students think, how they solve problems, and how their cognitive strategies develop over time. The core components of cognitively guided instruction include the following:

- **Elicits and analyzes student thinking** — Teachers pose open-ended problems and observe students' strategies, misconceptions, and approaches.
- **Uses student problem-solving strategies to guide instruction** — Instructional decisions flow from what students show they understand, not from a predetermined sequence. Teachers scaffold learning within a child's zone of proximal development.*
- **Allows for open-ended problem solving and discourse** — Students explain their thinking, examine others' reasoning, and revise their understanding.
- **Continuously embeds formative assessments** — Teachers adjust support based on real-time assessments and ongoing observation.
- **Provides instructional flexibility** — Teachers tailor tasks, scaffolds, and questions to align with students' developmental stages.
- **Creates a collaborative, inquiry-rich classroom culture** — Ideas are co-constructed as students engage in reasoning, discussion, and exploration.

Research suggests that CGI can have many benefits, but its implementation remains limited due to 1) limited research on early learners, 2) curriculum constraints, 3) lack of resources in some settings, and 4) the need for sustained professional development. These limitations suggest further study is needed, especially for early learners and academically struggling learners.

***Note**: *The zone of proximal development is the range of tasks a student can perform with support but cannot yet perform independently.*

When students are learning a new skill (or struggling to do so), explicit instruction is critical for building understanding.

Explicit instruction can be used across all grade spans to introduce new topics or review concepts students struggled to grasp. Explicit instruction includes teacher modeling and think-alouds, explaining and demonstrating specific strategies, carefully sequenced examples, and cumulative review. It must also include ample opportunities for guided practice for students, with feedback. It is an instructional approach that can be used throughout preschool through Grade 12 courses.

With explicit instruction, content is delivered in small chunks that build on mastered prerequisite skills or knowledge. When delivering explicit instruction, teachers should keep explanations concise and goal directed and connect content to underlying ideas and representations.

Studies find that explicit, step-by-step teaching of concepts and procedures improves performance in both computation and word-problem-solving. Direct instruction, one model of explicit instruction, demonstrates strong and durable effects across grade levels and subjects, including mathematics, by reducing unproductive cognitive load, allowing learners to focus on essential relationships and strategies rather than extraneous problem-solving demands.

Despite criticisms, explicit instruction is proven to have utility, especially with academically struggling students.

Explicit instruction has been criticized for being overly rigid, narrowly focused on fundamental skills and memorization, and often applied without sufficient teacher support. Its early use as a remediation tool for students of color from low-income households led some to label it a "racist program" for assuming these learners cannot engage in higher-order thinking. Critics also argue it becomes a "drill-and-kill" method that neglects the conceptual understanding students need for later math. Yet research shows that explicit instruction, when combined with strong teaching, curricula, and supports, improves outcomes for students who struggle with math, including those with learning disabilities. It should be part of a balanced instructional approach, not the only or primary approach.

A direct instruction curriculum significantly enhanced middle school students' fraction skills.

A study examined the effects of a direct-instruction curriculum, *Corrective Mathematics: Basic Fractions*, with 30 culturally and linguistically diverse seventh-grade students who had repeatedly failed the state math assessment and struggled particularly with fractions. Instruction occurred in small groups of six to seven students during 50-minute lessons that included 20 minutes of review and 30 minutes of new content.

	Pre-Test	Post-Test
Mean Performance	20%	77%
Score Range	0%-57%	36%-100%

Teachers introduced, demonstrated, and modeled each new skill before guiding students through practice; once students showed mastery, they moved on to independent work. Materials included scripted teacher lessons, student workbooks, and visual supports such as pictures and drawings. Students alternated between two days of this direct-instruction approach and two days of traditional instruction each week, with traditional instruction consisting of teacher demonstrations followed by practice using a remedial workbook aligned to the state test. ***Note****: Only three students scored lower than 75.*

As students learn new skills, it is important to reserve time for repetition and practice to build speed and fluency.

Repetition and practice purposefully help build fast and accurate recall of foundational facts (e.g., multiplication tables) and skills (e.g., fraction operations). Students may practice in low-pressure settings using games or flash cards to build speed and fluency.

Some teachers may also use short sprints of cumulative practice (e.g., timed tests); however, research indicates these can trigger math anxiety for some learners. Critics also suggest that the overuse of timed tests can limit students' mathematical proficiency and negatively impact their confidence as math learners.

Automaticity with arithmetic facts reduces a learner's cognitive load, frees up working memory, and reinforces their conceptual understanding and procedural fluency. Building fluency in fundamental facts and standard algorithms early enables students to easily transfer that knowledge to more novel, complex problems in later grade levels.

It is important to note that learners do not develop procedural fluency by repetition alone. Procedural fluency develops as students begin to understand number relationships and have learned multiple basic fact strategies that can be applied to various problems and contexts.

Repetition and practice also involve distributing learning and review sessions for a topic over time.

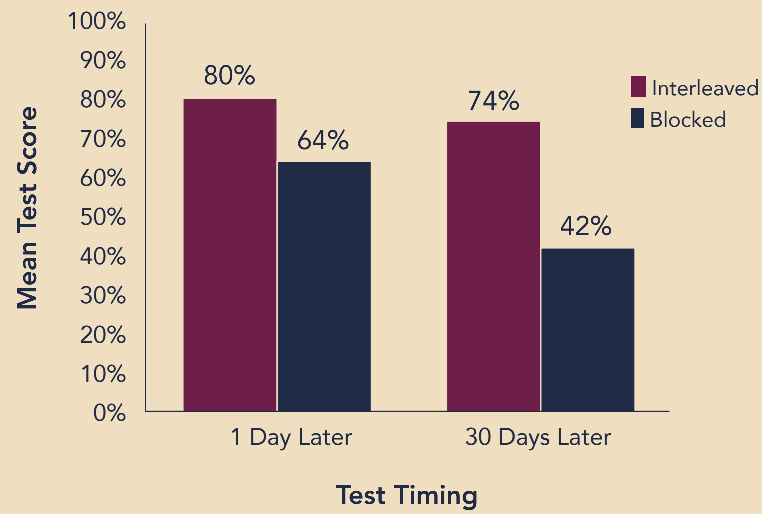
The distribution of learning and review sessions for a topic over time, also known as spacing, and using retrieval are among the most effective learning approaches in cognitive psychology and transfer well to math. This is especially true for strategic spacing, such as interleaved practice, which mixes different types of math problems, instead of giving students identical types of problems (e.g., all subtraction, all addition). Interleaving requires students to strategically choose the right strategy, which has been shown to improve retention and problem-solving.

Strategic repetition and practice can have an even greater effect on student learning.

A study tested whether interleaving different types of math graph and slope problems instead of grouping identical ones together (also known as blocking) significantly improved seventh graders' learning and retention. Students completed the same problems over three months, but half received them in an interleaved order. No student saw the same problem more than once; however, two groups of students received the same set of problems. After a review session, students took an unannounced test either 1 day or 30 days later. Interleaved practice consistently produced higher scores on both tests, demonstrating that the benefits did not fade over time.

Mean Test Scores after 1 and 30 Days of Review

Reproduced from Rohrer et al., (2014)



As students learn and then develop mastery of a skill, instruction shifts from concrete to abstract.

The CRA approach describes a typical instructional framework where students are gradually weaned off concrete materials as they become more comfortable with concepts, eventually applying abstract thinking to math problems.

Concrete
Students are introduced to a topic or concept with concrete materials (e.g., counters, coins, blocks, partitioned shapes).

Representational
Students become less reliant on manipulatives and gradually progress to pictorial representations (e.g., drawings or diagrams).

Abstract
Students have a secure understanding of the concept and can rely on abstract symbols (e.g., numbers, symbols, formal mathematical notations) to problem solve.

Benefits of the CRA Approach in Mathematics

1. Helps students make sense of ambiguous abstract symbols.
2. Enables students to map abstract symbols to physical experiences or representations of those experiences.
3. Equips students with a bank of stored images that can be retrieved when abstract symbols lose meaning.
4. Guides students to move beyond surface features of concrete materials, enabling the transfer of knowledge to novel contexts.

CRA is often conceptualized as a sequence; however, recent research emphasizes that conceptual understanding develops as students move flexibly among concrete materials, visual models, and symbolic notation, often using more than one at the same time. Studies of CRA-integrated interventions, in which manipulatives, drawings, and symbols are taught simultaneously and then gradually faded, show strong gains in fractions, number sense, and place value for students with learning difficulties. CRA-integrated student outcomes were comparable to or better than traditional sequential CRA.

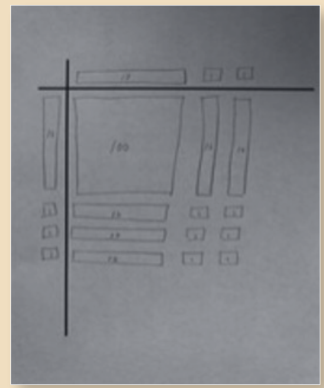
Students build on each stage of learning until they develop abstract reasoning.

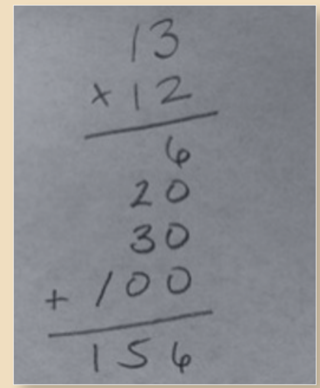
Research indicates CRA is an effective approach that can help students bridge the gap between conceptual and procedural knowledge, especially for students with disabilities.

Though often associated with early childhood and elementary students, concrete and representational thinking can also benefit middle and high school students.

One study of high school algebra students with learning disabilities found that the use of manipulatives increased students' conceptual understanding of square roots.

Students use physical objects, such as manipulatives, to develop a concrete understanding of new math topics. A meta-analysis of 55 studies found small to moderate benefits for instruction with manipulatives compared with instruction using only abstract math symbols. The example below illustrates how students move from concrete to abstract thinking in multi-digit multiplication.





Concrete	Representational	Abstract
Students use manipulatives to represent the numbers and complete the task.	Students create a picture representing the same numbers to complete the task.	Students model the problem using numbers and symbols to complete the task.

Theme 2

What Math Skills, Content, and Courses Are Taught

ECE and K-12 math instruction in the U.S. generally follow a standard sequence over multiple grade spans.

A well-designed early childhood and K-12 math sequence builds durable proficiency by moving from concrete number sense in the primary grades to generalized, abstract reasoning in high school. Those who do research and write standards emphasize that progression is not arbitrary: Topics are ordered to reflect how mathematical ideas *develop in learners* and how they *cohere within the discipline*.

It is helpful to examine the math instructional sequence in two stages: early foundations and algebra and beyond.

Early Childhood (Ages 3-5)	**Lower Elementary (Grades K-2)**	**Upper Elementary (Grades 3-5)**	**Middle School (Grades 6-8)**	**High School (Grades 9-12)**
Foundations of Number Sense	Building Number Sense and Operations	Procedural Fluency, Fractions, and Expanding Operations	Pre-Algebra and Proportional Reasoning	Algebra to Advanced Mathematics

EARLY FOUNDATIONS

The developmental stage when children construct the core relationships among numbers, begin to develop geometric and spatial reasoning, and understand measurement. Children learn how to explain, represent, and apply these ideas.

ALGEBRA AND BEYOND

The stage of mathematics learning where students move from concrete arithmetic and spatial reasoning toward abstract, relational thinking.

This sequence is intended to help students build mathematics content knowledge and skills progressively.

Early Childhood At this stage, children develop informal mathematical understanding through play and daily routines.	• **Counting and Cardinality**: Reciting numbers in order, counting objects accurately, and understanding that the last number counted tells "how many." • **Quantitative Comparison**: Using words like "more," "less," and "same" and recognizing which set has more. • **Measurement and Sorting**: Comparing sizes, lengths, and weights; sorting objects by color, shape, or size. • **Pattern Recognition**: Identifying and creating simple repeating patterns (ABAB, AABB). • **Spatial Awareness**: Understanding positions (e.g., above, below, up) and basic shapes.
Lower Elementary This stage establishes the skills, knowledge, and concepts for computation and algebraic thinking.	• **Number Sense**: Counting to 100+, skip counting, and understanding place value. • **Addition and Subtraction**: Fluently adding or subtracting numbers up to 20 using strategies such as counting on, making 10 (e.g., creating a group of 10 to add to), and doubles facts (e.g., 4 + 4, 5 + 5). • **Introduction to Multiplication Concepts**: Early understanding of repeated addition, equal groups, and arrays. • **Measurement and Data**: Using rulers, clocks, and coins; reading simple graphs. • **Geometry**: Recognizing and describing 2D and 3D shapes; partitioning shapes into halves and quarters. • **Problem Solving**: Using addition and subtraction in various contexts such as word problems.
Upper Elementary Students develop abstract thinking related to more complex operations.	• **Multiplication and Division**: Demonstrating fluency with basic multiplication and division facts. • **Fractions and Decimals**: Understanding fractions as numbers, comparing fractions, adding/subtracting fractions with like/unlike denominators, and relating decimals to fractions. • **Multi-Digit Operations**: Performing addition, subtraction, multiplication, and long division with larger numbers. • **Measurement Conversions**: Converting within the same system (inches to feet, minutes to hours). • **Geometry**: Understanding area, perimeter, volume; classifying shapes by properties. • **Data and Graphing**: Understanding bar graphs, line plots, and introduction to coordinate planes. • **Introduction to Variables**: Building awareness of simple expressions and equations.

Math skills are taught sequentially because advanced skills are scaffolded on knowledge and fluency of prior skills.

Although there is relatively broad agreement on what students need to learn in early childhood through upper elementary school, more debate emerges in middle school and especially in high school.

Middle School The emphasis shifts to algebraic thinking and formal problem-solving.	• **Ratios, Rates, and Proportions**: Solving real-world proportional problems and unit rates. • **Integers and Rational Numbers**: Performing operations with positive/negative numbers and fractions. • **Expressions and Equations**: Simplifying expressions and solving one- and multistep equations and inequalities. • **Geometry**: Understanding of volume and surface area of solids, transformations (translations, rotations, reflections), and the Pythagorean theorem. • **Statistics and Probability**: Understanding measures of central tendency, variability, and probability models. • **Functions**: Understanding the concept of input-output relationships.
High School Students deepen their conceptual understanding and prepare for postsecondary or career applications.	• **Algebra I**: Solving linear equations, inequalities, systems, quadratic functions, and basic polynomials. • **Geometry**: Understanding proofs, congruence, similarity, circles, and trigonometric ratios. • **Algebra II**: Understanding complex numbers, exponential/logarithmic functions, advanced polynomials, and sequences and series. • **Precalculus**: Understanding functions, identities, and advanced function analysis. • **Calculus**: Understanding derivatives, integrals, limits, and applications. • **Probability and Statistics**: Solving advanced data analysis, normal distributions, statistical inference, and data • interpretation. • **Modeling and Applied Math**: Using mathematics to represent and solve real-world problems.

However, there is no consensus about what high school course sequences best help students build advanced math skills.

The sequence of high school math courses varies. Some students complete some traditional high school courses while in middle school. Furthermore, not every student takes every high school math course as some are required while others are offered as electives. The courses students actually take are often dependent on their academic track and postsecondary plans.

While it is clear that students need to build advanced mathematical skills that set them up for success in rigorous postsecondary coursework and math-dependent careers, there is less clarity over the best course pathways to do that. The two main questions at the root of this debate are:

1. **What should students' final, or "capstone," math course be?**

2. **Would an integrated pathway better support students to master high-level mathematical skills and concepts?**

A traditional math sequence terminates with one of three capstone course options.

Traditional Math Sequence

GRADE 8	GRADE 9	GRADE 10	GRADE 11	GRADE 12 (Capstone)
Grade 8 Math	Algebra I	Geometry	Algebra II	Precalculus
Grade 8 Math	Algebra I	Geometry	Algebra II	Statistics
Algebra I	Geometry	Algebra II	Precalculus	Calculus

Each potential capstone course has benefits and drawbacks related to students' postsecondary options and long-term outcomes.

PRECALCULUS

Completing precalculus improves students' college readiness compared with stopping at Algebra II. However, students whose highest K-12 course is precalculus have lower college-going and bachelor's degree completion rates than those who take Advanced Placement (AP) Statistics or AP Calculus, but higher than students who complete non-AP statistics.

STATISTICS

The long-term earnings for students who take AP Statistics are about equal to students who take AP Calculus AB. This course also offers strong alignment with 21st century quantitative needs and is relevant for students pursuing social science majors. However, it provides weaker preparation for STEM majors. Importantly, non-AP courses may be poorly designed, lacking the same college prep and long-term benefits as AP Statistics.

CALCULUS

Calculus is the strongest predictor of entering and persisting in STEM majors; it also opens doors to selective institutions of higher education. However, concerns exist regarding racial and socioeconomic disparities in access to calculus. Moreover, a calculus-first curriculum leaves little room for modeling, computation, or data science — all of which are vital skills in today's economy.

The capstone course a student completes has implications for their postsecondary options.

High School Mathematics Capstone Course

	PRECALCULUS	AP STATISTICS	AP CALCULUS
College Access	More likely to enroll in a four-year university than students who only complete Algebra II, but less likely than peers who complete AP Statistics or AP Calculus.	More likely to enroll in a four-year university than students who complete precalculus, but slightly less likely than peers who complete AP Calculus.	Most likely to enroll in a four-year university and a selective institution of higher education.
Undergraduate Major and Career Options	Precalculus does not close off STEM majors but often means students will need to start in college precalculus or calculus, and they are competing against peers who already took AP Calculus.	Well represented in math-intense majors and careers, but somewhat more likely to major in business, finance, or information, than STEM. May be somewhat less likely to be accepted into selective institutions due to beliefs about the rigor of AP Statistics versus AP Calculus.	Substantially more likely than peers to major in STEM, including engineering, physical sciences, computer science, or math. Also more likely to enroll in selective institutions of higher education.

> *"The highest level of mathematics reached in high school continues to be a key marker in precollegiate momentum, with the tipping point of momentum toward a bachelor's degree now firmly above Algebra 2."* —CLIFFORD ADELMAN, 2006

Note: *Course pathways are not a one-size-fits-all and that a student's pathway will depend on various factors including demonstrated proficiency and interests.*

Beyond the capstone question, there is a push to rethink the math sequence entirely — moving to an integrated approach.

In an integrated mathematics course sequence, students would progress through courses (e.g., Math I, Math II, Math III) that offer a **mix of algebra, geometry, and statistics** using spiral learning (revisiting and building upon concepts over time), rather than teaching mathematical strands in isolation.

There are a number of potential benefits to an integrated approach …	… but there are also some important considerations.
• Promotes conceptual coherence across mathematics domains. • Encourages repeated exposure to key ideas over multiple years. • Supports equitable access by keeping all students in a shared pathway for longer time frames. • Aligns more closely with high-performing international models (e.g., Singapore, Japan). • Allows early integration of statistics and data science, supporting modern quantitative literacy and modeling.	• Implementation requires careful curriculum design and reorganization of course structures, supported by significant teacher professional learning and high-quality materials. • Structure and progression are less clear; may create communications challenges with families and other stakeholders and could result in less clarity on transcripts about what courses a student has completed (i.e., "Math III" versus "Calculus").

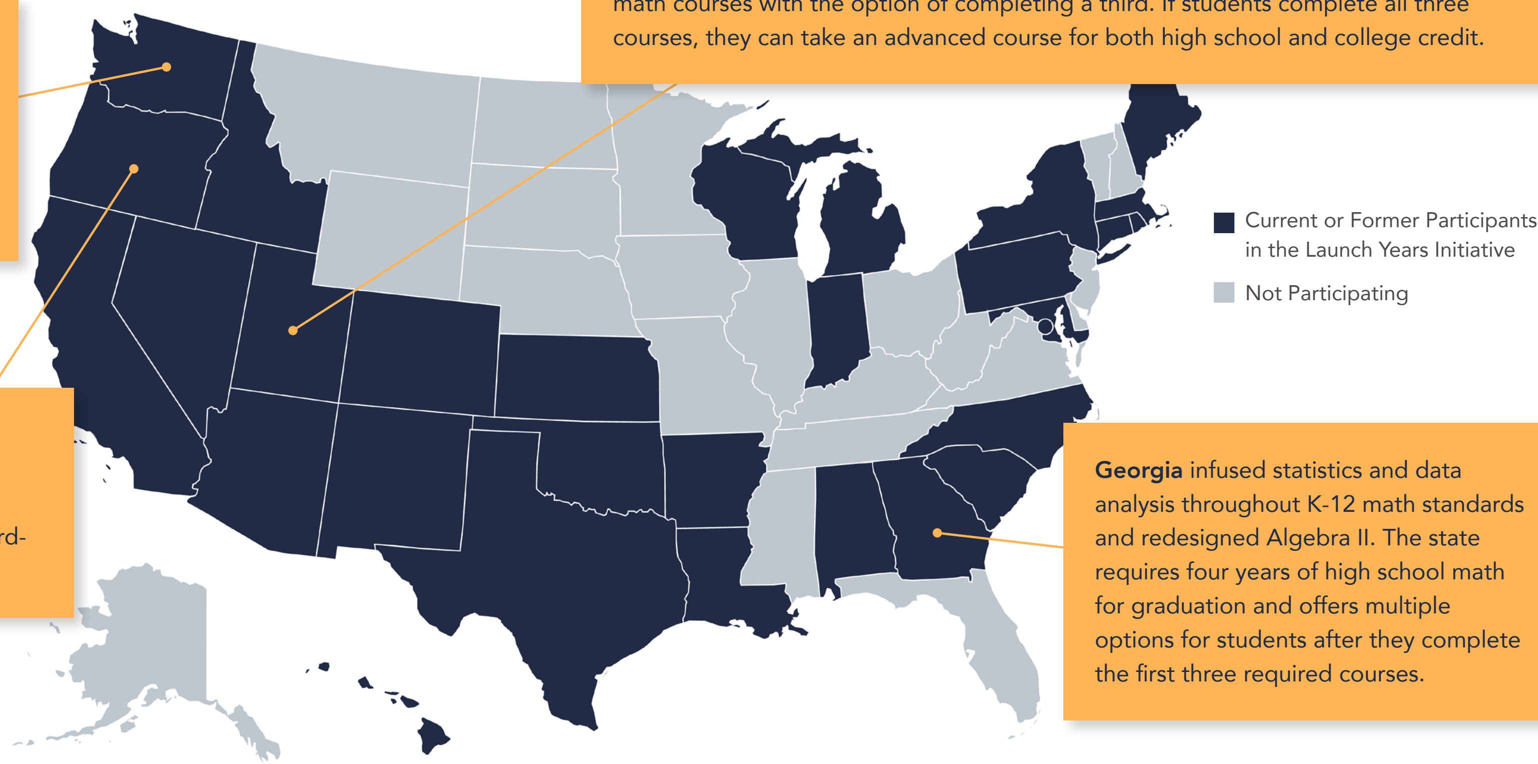
Between 2018 and 2026, 30 states have focused on updating high school math pathways, including integrated pathways.

The Charles A. Dana Center's Launch Years Initiative is a multi-state effort to redesign the transition from high school to postsecondary education by promoting multiple math pathways aligned with students' aspirations and workforce demands. States commit to revising graduation and college admission requirements, modernizing course sequences, supporting educators, and improving advising.

Current and Former Participants in the Launch Years Initiative

Utah adopted an integrated high school core sequence and created multiple pathways for students. Under the new model, students take at least two years of integrated math courses with the option of completing a third. If students complete all three courses, they can take an advanced course for both high school and college credit.

Washington revamped its Algebra II courses, adding units on data science, quantitative reasoning, and mathematical modeling, to prepare students for multiple postsecondary pathways.



Oregon replaced an "Algebra II for all" approach with two years of integrated, foundational math followed by student-selected third- and fourth-year options.

Georgia infused statistics and data analysis throughout K-12 math standards and redesigned Algebra II. The state requires four years of high school math for graduation and offers multiple options for students after they complete the first three required courses.

Theme 3

Who Receives What Instruction

As students progress from early childhood to high school math, gaps can accumulate and instructional needs diverge.

Although math instruction generally follows a standard sequence, students' needs will vary as instruction and learning varies. To address learning gaps, help students achieve proficiency, create opportunities for acceleration, and support students in their long-term aspirations, schools adopt several strategies for targeting instruction, including the following:

Between-Class Differentiation, or "Tracking" — Students are placed into accelerated, regular, or remedial courses and taught in separate classrooms for one or more subjects. Math tracking often starts in seventh to ninth grade, and determines when students take Algebra I.

Within-Class Differentiation — Teachers assess students' learning needs and tailor instruction accordingly. "Ability grouping" is one approach that often starts in elementary school and may precede between-class differentiation in middle school.

Competency-Based Education — Students progress through math content based on when they have demonstrated competency of a given concept and readiness to advance. It can start at any time and can be used to supplement within- or between-class differentiation and include below- or above-grade level material.

Differentiation is intended to help:

- Fill gaps in student learning that may prevent them from mastering new material.
- Adapt the pace and depth of instruction to students' needs and abilities.
- Offer opportunities for accelerated learning.

Between-class differentiation sparks questions about the benefits of tracking students based on learning needs.

Arguments *in Favor of* Tracking

- Students learn more effectively when grouped with peers at similar levels; some students need accelerated content to continue to challenge themselves and others need a different type and slower pace of instruction to continue learning.

- Teachers often struggle to differentiate instruction in classrooms where proficiency varies widely; creating classrooms that group students according to their ability and/or achievement makes it more feasible to do so.

> *"Acceleration promotes academic development by moving students at an educational pace matching their abilities and readiness for advanced curriculum rather than restricting them to a set grade-level curriculum based on their age."* —NATIONAL ASSOCIATION FOR GIFTED CHILDREN, 2025

Arguments *Against* Tracking

- Tracking reproduces and amplifies racial and socioeconomic inequities, as students from low-income backgrounds, Black students, and Hispanic students are less likely to be assigned to accelerated tracks than their high-income, white, and Asian American peers.

- Classrooms that include students in the lower tracks have lower-quality teachers and deliver systematically weaker instruction and expectations, often to students who are most in need of support.

> *"Too often, it is not ability, but student characteristics (such as race, wealth, and privilege) and/or school-based resources (such as instructional resources, placement practices, school culture, and teacher and school counselor behavior) that contributes to the stratification of higher-level learning opportunities by race and income."* —*OPPORTUNITY DENIED*, 2023

Research on the effects of tracking on academic and nonacademic math outcomes is mixed.

Some Research Has:	For Example:
Found positive effects of tracking on students' future test scores.	One study of data from the National Education Longitudinal Survey found that eighth-grade tracking positively affected 12th-grade standardized test scores in mathematics.
Found no or mixed effects of tracking on students' future test scores.	One study found increased achievement for students identified for advanced classes based on prior achievement, but no effect on those identified based on ability tests; benefits were concentrated among low-income students and Black and Hispanic students.
Examined the effects of detracking on student test scores.	One study examined a program in which students at or below grade level were placed in an Algebra I class in ninth grade, when some students would have otherwise been placed in ability-grouped courses. Teachers received additional resources and professional development. The study found positive effects on math achievement.
Found negative effects on students' self-perception.*	Some research shows that placement in advanced math courses is associated with lower student self-perceptions of competence and lower student confidence that they will graduate from high school.
Found effects of tracking on students' sense of belonging.*	Some research shows that being assigned to advanced tracks in math increased students' perceptions of their belonging at school and how they incorporate academic success as part of their personal identity.

***Notes**: *Although there are theoretical ties between student perceptions, self-concept, and engagement and student academic performance, most research stops short of making connections between nonacademic and academic outcomes.*

How courses are designed and staffed play a critical role in student experiences and learning.

Tracking Design Feature:	For Example:
How Teachers Are Assigned	How teachers are assigned to tracked classes and prepared to teach advanced or remedial content affects the quality of instruction. Some research on within-school teacher assignments in a large urban district shows that students in nonadvanced courses are assigned teachers who are more likely to be less experienced, minority, and female compared with those in advanced courses.
When It Starts	When tracking starts and how it creates path dependency for future coursework affects how math learning accumulates, and the types of coursework students are prepared to take in high school and beyond. There is generally more consensus that tracking in elementary school is too early; as a result, most tracking — and most of the debate about tracking — is anchored in middle and high school.
How Students Are Identified	Students are often tracked based on parent or teacher referrals, ability test scores, or past achievement on standardized tests. Parent or teacher referrals can introduce nonacademic factors into tracking decisions and can lead to under-identifying low-income, Black, and Hispanic students for advancement; ability and achievement tests are more objective, though also imperfect (e.g., if they depend on language ability). Auto-enrollment policies, in which students are enrolled in advanced courses based on past achievement — without having to “opt in” — are often a preferred way to determine students’ tracks.
Path Dependency	How tracking affects future coursework and long-term outcomes depends on whether students have one or multiple opportunities to “jump tracks” or be reassigned to a different track as their learning progresses.

Within-class differentiation is another strategy for targeting instruction to students' needs and abilities.

As opposed to tracking, which differentiates instruction by moving students into different classes, teachers who implement within-class differentiation tailor instruction to the variety of learning needs within their classroom. This is commonly understood to include differentiation on at least three dimensions:

Content	Process	Products
For example, re-teaching skills or concepts that address learning gaps or using visuals or manipulatives to aid learning.	For example, ability grouping in which some students receive small-group instruction while others work independently.	For example, allowing students to demonstrate learning through an essay or presentation.

Within-class differentiation tends to generate less debate than tracking, in part because it does not raise the same issues of design and implementation:

How Teachers Are Assigned	Students' teachers remain the same, as teachers maintain classrooms of students with mixed needs and abilities.
When It Starts	Teachers can decide when to differentiate instruction and when not to, which could vary for each lesson.
How Students Are Identified	Within-class differentiation may be based on short-cycle assessments, teachers' observations of learning, or student choice.
Path Dependency	Teachers can group and regroup students on an ongoing basis, according to dynamic student needs.

Research suggests within-class differentiation can improve student learning, but design and implementation matter.

Within-class differentiation is grounded in theory and research, with some mixed outcomes:

- Research on the elements of within-class differentiation — that is, adapting instruction based on student readiness, interest, learner profile — provides a basis for its use.
- One study of ability grouping from the 1980s found that "within-class ability grouping in mathematics is … found to be instructionally effective" with slightly larger effects for low achievers than for average or high achievers.
- A meta-analysis found that the average benefits from ability grouping masked variation in effect sizes.

Although research generally finds positive effects on student learning, within-class differentiation depends on design and implementation:

- Research suggests that the effectiveness of differentiated instruction within a classroom depends on teachers' skill and often requires professional learning.
- Investigations into the implementation of differentiated instruction suggest that design features, such as planning time, are important.
- Differentiated instruction is bolstered by the effective use of assessment data to understand what students have learned and where they have gaps.

Competency-based education can also help ensure students receive differentiated instruction …

Competency-based education (CBE) is an approach to personalized learning in which students advance only after demonstrating what they have learned. It is often aided by data and technology that assess individual student learning needs and objectives. It can be used to supplement or supplant within- or between-class differentiation, as well as to address both below-level learning gaps and teach above-level material.

Addresses Learning Gaps

CBE helps prevent learning gaps because students advance only after demonstrating readiness. It also helps address learning gaps by identifying and revisiting the specific skills and knowledge students are missing.

Self-Paced Advancement

CBE enables students to progress toward and beyond proficiency at their own pace, not only reducing learning gaps but also allowing students to move on to the next competency or beyond grade-level material when ready.

Clear Goals and Assessments

Articulates clear learning goals and enables aligned assessments that teachers and students can use to understand where targeted interventions or acceleration will support students' continued learning.

CBE has been the subject of academic research, but the evidence of its effectiveness is nascent — in part because it is difficult to isolate competency-based approaches from other efforts to personalize education.

… but it faces many barriers to implementation.

Identifying Competencies

CBE requires finding the right "grain size" for competencies. Too large a grain size may let learning gaps sneak in; too small, and competencies can lose coherence as part of broader learning progressions.

The Role of the Teacher

CBE requires rethinking the role of classroom teachers, often requiring professional learning and the use of more data; it also has implications for teacher credentialing.

Change Management

CBE requires districts and schools to engage stakeholders (e.g., students, parents, teachers) in the design and adoption process, as well as political support from governing entities.

School Structures and Processes

CBE requires challenging the "grammar of schooling" — from bell schedules to seat time, graduation requirements, and how colleges understand and assess high school transcripts.

Accountability Policies

CBE requires redesigning annual summative assessments that focus on seat time and age-based cohorts to enable and encourage competency-based approaches.

Instructional Materials

CBE implementation requires curricula and materials that enable students to progress when they are ready.

The challenges of implementation extend beyond efforts to improve math education via competency-based learning.

Contents

Four major barriers often stand in the way of improving math performance in states and school districts.

Educator Preparation Policies

Instructional Materials and Professional Development

Culture of Math Anxiety

State Policy Environments

Inadequate educator preparation hinders math instruction, particularly in the early grade levels.

Teacher preparation programs often require little math coursework and pedagogical training, leaving educators underprepared to teach critical concepts such as fractions, place value, or algebraic thinking. This is especially a challenge at the elementary level, where math instructional requirements can already be more limited.

Only 21 states set math standards for teacher preparation that cover all necessary elementary knowledge domains.

Just 3% of elementary teachers have a degree in math or math education, compared with 45% of middle school teachers.

The average graduate program dedicates only 14 hours of instructional time to foundational math content knowledge.

Arkansas is one of the few states that require educator preparation in both math content and pedagogy, with emphasis on conceptual understanding and procedural fluency.

> *"When you're doing teacher prep for teachers who are going to teach reading, you don't have to teach them how to read, but when you're doing teacher prep for teachers of math, you have to teach them math in addition to how to teach math … you almost need to have double the focus on math."* —SARAH POWELL, PROFESSOR, UNIVERSITY OF TEXAS AT AUSTIN

The uneven implementation of HQIM and aligned professional development in math poses a challenge.

The adoption of HQIM in math remains limited across states. A recent survey by RAND found that 55% of math teachers report regularly using at least one standards-aligned instructional material in their classrooms (compared with 44% of English language arts [ELA] teachers); however, they often adapt and supplement their curriculum with other materials that are often not high quality or aligned with standards (e.g., resources from the internet).

A lack of robust math-specific professional development and coaching supports undermines HQIM implementation. According to NCTQ, "28 states provide funding for professional learning for in-service teachers in math, and only 6 explicitly align these opportunities with HQIM implementation."

Math teacher professional development can often entail one-off, "sit and get" workshops rather than sustained, collaborative learning opportunities. Without sufficient time to plan, practice, and receive coaching, teachers revert to familiar teaching habits that undercut efforts to improve student learning. This is particularly a problem because professional development in math tends to be less effective than professional development in other subjects if it is not sustained, practice based, and directly tied to the curriculum. Few schools and systems invest in extensive math coaching.

> *"Districts could spend huge amounts of money on the best curriculum, but if teachers aren't supported in how to use it, if they don't have training, if they don't have time to dig in and ask questions, it won't matter what was purchased."* —EBONEY MCKINNEY, PRESIDENT, ASSOCIATION OF STATE SUPERVISORS OF MATHEMATICS, AND DIRECTOR OF MATHEMATICS AND EDUCATIONAL TECHNOLOGY, ARIZONA DEPARTMENT OF EDUCATION

A culture of math anxiety significantly reduces student learning opportunities and performance in math.

About one-third of young people have never considered themselves a "math person." Student math identities start forming as early as kindergarten and tend to crystallize by the end of elementary school, just as courses start to differentiate. High levels of math anxiety are consistently linked with lower achievement, reduced willingness to take advanced courses, and avoidance of math-related college majors and careers.

Teachers can transmit their own math anxiety to their students, with negative effects on student achievement and the amount of instructional time spent on math. Approximately one in four elementary teachers reports math anxiety.

Parent math anxiety is negatively associated with the learning outcomes and math confidence of their children, including in pre-K.

Math anxiety is more common among female students and racial minorities, reinforcing inequities in opportunity and achievement.

NAEP fourth-grade math proficiency rates of students with...

Low Math Confidence **11%**

High Math Confidence **63%**

States often lack coherent policy focus and consensus around evidence-based practices.

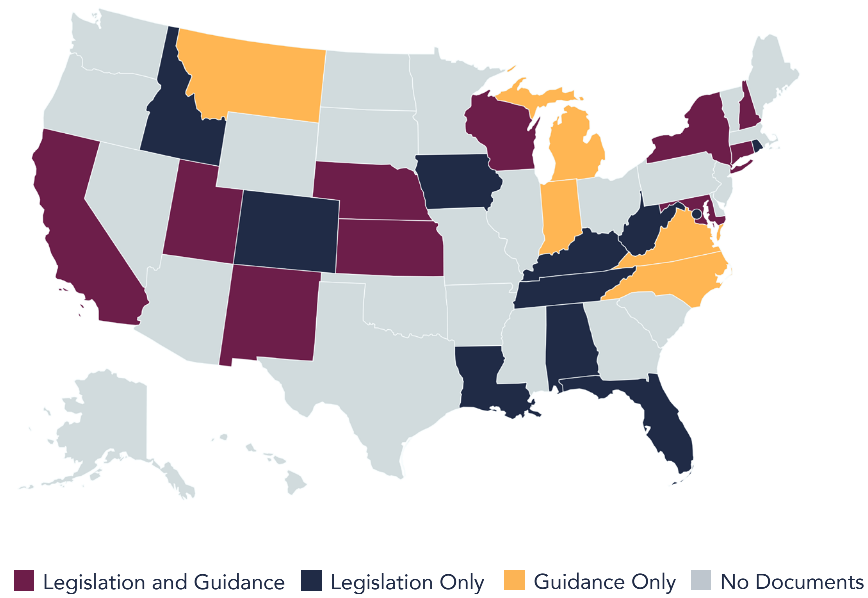
Lack of Policy. Only 27 states have laws or guidance related to math instruction, leaving about half of the country without identifiable math-specific policies.

Lack of Coherence. Math standards and policies can be contradictory. For example, some call for explicit teacher-led instruction while also requiring student inquiry and problem-based learning, with little clarity on how to balance or sequence these strategies.

Lack of Focus. State standards can be very broad and wide-ranging, pressuring teachers to cover too many discrete skills at the expense of deeper conceptual work. For instance, one analysis found that a student's math education leading to Algebra I may include 150 discrete concepts and skills, but only a fraction of those are key predecessors to success in the subject.

Lack of Evidence. While many state policy documents refer to "evidence-based practices," they often do not define what counts as evidence, creating space for trendy but weakly supported approaches to take hold.

States With Math Guidance or Legislation Documents



27 states have policies related to core instruction

14 states have policies related to screening and identifying students for intervention

16 states have policies related to intervention for struggling students

Contents

Efforts to strengthen math teaching and learning across the country have deep roots that extend over 100 years.

U.S. Math Policy Milestones, 1894-1983

1894
The National Education Association's *Committee of Ten* report advocates for a single high school curriculum in which all students take advanced math in preparation for college.

1900s-1930s
Progressive education movement leads to greater emphasis on play- and discovery-based approaches to learning, including in mathematics.

1918
The *Cardinal Principles of Secondary Education* report, released by the National Education Association, advocates for differentiated tracks in high school and is followed by major declines in advanced math course-taking rates.

1923
The *Reorganization of Mathematics in Secondary Education* report by the Mathematical Association of America attempts to lay out a coherent national program and sequence of math teaching and learning in Grades 7-12.

1940s-1950s
The Life Adjustment Curriculum movement aims to target education to the "middle 60%" of students, leading schools to move away from traditional mathematics courses and toward applied courses like business arithmetic, personal finance, and "life math."

1957-1958
The launch of Sputnik and passage of the National Defense Education Act precipitate large federal investments in math and science education, a reemphasis on calculus in high school, and a move away from progressive approaches.

1960s-1970s
The "New Math" movement, supported by federal research funding, leads to a major overhaul of curricula and instructional materials, driving a deeper focus on conceptual understanding and abstract thinking. "Math wars" between traditionalists and progressives erupt, and a "back-to-basics" movement reemerges.

1965
The Elementary and Secondary Education Act (ESEA) establishes large-scale federal aid to schools serving students from low-income families, making the improvement of fundamental reading and math achievement a central federal concern.

1983
The National Commission on Excellence in Education releases *A Nation at Risk* and warns of a "rising tide of mediocrity" in American education, highlighting weak course-taking and performance in math as a threat to the country's economic prosperity and national security.

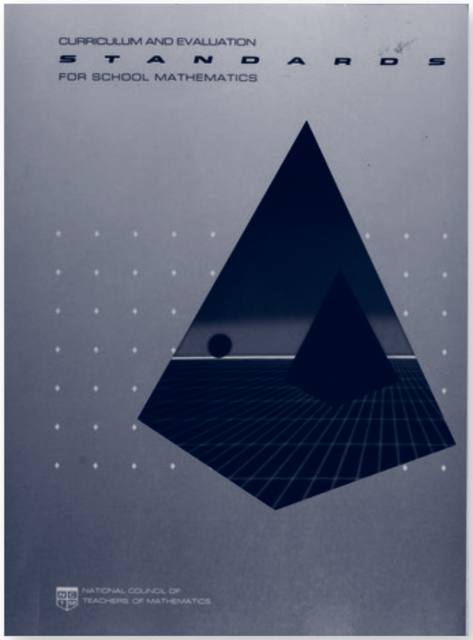
Since the late 1980s, several overlapping themes have shaped math policy and instruction.

Theme	Approximate Years	Key Developments
The Math Wars	1988-2000	Publication of early math guidelines by the National Council of Teachers of Mathematics (NCTM); math wars and disagreements over changing standards
Standards-Based Reforms	1990-2000	State and federal efforts to strengthen academic standards and systemic coherence
High-Stakes Testing and Accountability	2002-2013	No Child Left Behind (NCLB); national focus on improving math proficiency, especially in elementary and middle schools
National Standards Era	2010-2016	Adoption of CCSS and subsequent implementation challenges
State-Led Reform and Recovery Era	2015-2025	Every Student Succeeds Act (ESSA); COVID-19 pandemic; academic recovery efforts; high school math pathways reforms

Over these decades, the vast majority of math policy change has focused on the K-12 level. However, limited efforts to improve ECE math have also been present, often as part of broader pushes to improve the overall academic quality of the ECE sector.

The modern math reform movement can be traced to the publication of NCTM's national math guidelines in 1988.

NCTM's seminal *Curriculum and Evaluation Standards for School Mathematics* represented the first significant set of national math guidelines in the U.S. and reinforced a wider push around standards-based reforms.



The guidelines advocated for a starkly different approach to teaching mathematics in the U.S. They emphasized conceptual learning and problem-solving, played down the value of rote memorization and pen-and-paper computation, and reflected a constructivist approach to learning.

Throughout the 1990s, the National Science Foundation funded the development of curricula aligned to NCTM's vision, and some states set out to revise their math standards in alignment with the recommendations. A 1999 panel convened by the U.S. Department of Education endorsed several math curricula and programs reflective of the new guidelines.

> *"The recommendations [in these guidelines] present a vision of mathematics education vastly different from that now experienced by most students. ... The shift in emphasis is from an instruction system seemingly obsessed with computational skills to one in which these abilities are considered in the context of a broader interpretation of mathematical knowledge."* —NCTM, 1988

The 1990s saw persistent math wars amid a wave of standards-based reforms.

As states created new standards and schools began implementing new curricula and instructional approaches aligned with the NCTM guidelines, fierce math wars erupted between reform advocates who championed the guidelines and traditionalists who favored existing approaches.

Grassroots parent groups formed to protest the new standards. These groups often mobilized through early internet websites and expressed concern about declining math scores and the struggles of educators implementing new curricula and teaching approaches.

In 1999, a group of mathematicians and scholars, including Nobel laureates, signed an open letter to then-U.S. Secretary of Education Richard Riley protesting the Department's endorsement of NCTM-aligned curricula.

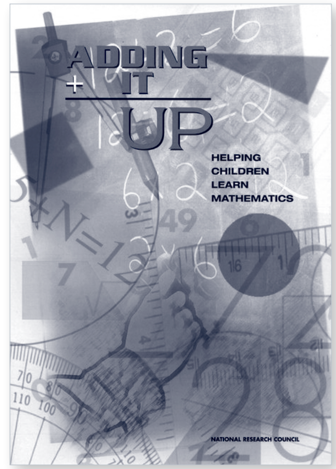
In the face of implementation challenges and ongoing public pressure campaigns, states and districts that had initially embraced the NCTM guidelines began to reverse course by the end of the decade. California, the largest state by population and one with significant sway over textbook publishing, had adopted new NCTM-inspired standards in 1992 but then backtracked in 1997 and adopted revised standards that returned greater emphasis to traditional skills and methodological approaches.

By the end of the 1990s, there was growing consensus that math standards should reflect a balanced approach.

In 2000, NCTM revised its math guidelines with the release of *Principles and Standards for School Mathematics*. In 2001, the National Research Council (NRC) published its seminal report *Adding It Up: Helping Children Learn Mathematics*.

Both reports set clearer expectations for procedural fluency and rote memorization in math instruction and advocated for greater balance between competing visions of mathematics. The NRC report advocated explicitly for balanced instruction and coined the term "mathematical proficiency," which combined various capabilities: conceptual understanding, procedural fluency, strategic competence (problem-solving), adaptive reasoning (logic), and productive disposition.*

This new advocacy for consensus and balance quieted the math wars and set the tone for the next decade.



"[One claim] is that mathematics is bound by history and culture, that students learn by creating mathematics through their own investigations of problematic situations. … A countervailing claim is that mathematics is universal and eternal, that students learn by absorbing clearly presented ideas and remembering them. ... The trouble with these claims is not that one is true and the other false; it is that both are incomplete." —NRC, 2001

Note: **This conception of "mathematical proficiency" continues to be cited in current debates about a potential Science of Math.*

During the math wars of the 1990s, states developed K-12 standards and performance measurement systems.

In 1994, federal policymakers enacted two laws that encouraged states to improve standards, assessment, and accountability.

- **The Goals 2000: Educate America Act** established improved math performance as one of several national education goals. It provided grants to states to develop academic standards, aligned assessments, and improvement plans.
- **The Improving America's Schools Act** required states to adopt academic standards, aligned assessments, and accountability systems as a condition of receiving federal ESEA funds. Enforcement was light and implementation was uneven.

These policies built on state momentum in standards-based reform and generally drew bipartisan support: Those on the right emphasized economic competitiveness, while those on the left emphasized educational equity and accountability for disadvantaged students. By 2000, nearly every state had implemented academic standards in math and reading or was in the process of implementing them. Some states, such as Texas and North Carolina, also held schools accountable for student performance.

Major research reports found that math performance increased in the 1990s and that this improvement was likely due to states' embrace of standards-based reform policies. Stories about states experiencing rapid gains in academic achievement also captured the attention of national news media. One of these stories, dubbed the "Texas Miracle," factored heavily into the 2000 presidential election, in which education was ranked as the No. 1 issue for voters.

NCLB held K-12 schools accountable for math performance and galvanized efforts for proficiency.

Enacted in 2002, NCLB pushed states to set stronger grade-level mathematics standards, develop aligned assessments to measure student proficiency, build data systems to track and report performance, and intervene to support struggling math learners.

State Requirements Under NCLB				
Annual standardized testing in math and ELA for all students in Grades 3-8 and once in high school	Adequate Yearly Progress (AYP) proficiency goals and accountability for meeting those goals	Publicly shared school report cards with performance disaggregated by student subgroup	Sanctions for schools that did not meet their AYP targets, including mandatory school choice, free tutoring, or restructuring	New requirements for teachers to be "highly qualified" — often demonstrated through educational credentials and subject matter competency

The law made boosting math achievement a highly visible and high-stakes national priority.

NCLB test-based accountability made math instruction more standardized, data-driven, and outcomes-focused.

Instructional Time: Schools and districts devoted more instructional time to math, particularly in elementary school.

Structured Instruction: Schools and districts introduced structured curricula, pacing guides, and interim assessments to improve coverage of state standards, monitor student progress, and target supports to struggling learners. This often helped students master fundamental skills but could pose challenges in supporting students ready for acceleration.

Triaged Student Supports: Schools implemented targeted interventions (e.g., math tutoring after school, pull-outs, remedial groups) for students at risk of failing their math exams. Schools focused intensely on supporting "bubble" students — those just below the proficiency cutoff — to boost their accountability ratings. This "triage approach" helped some previously overlooked students receive more attention and support. It also meant that the highest-achieving students and the lowest-achieving students received less attention because NCLB did not recognize schools for fostering student growth above or below proficiency.

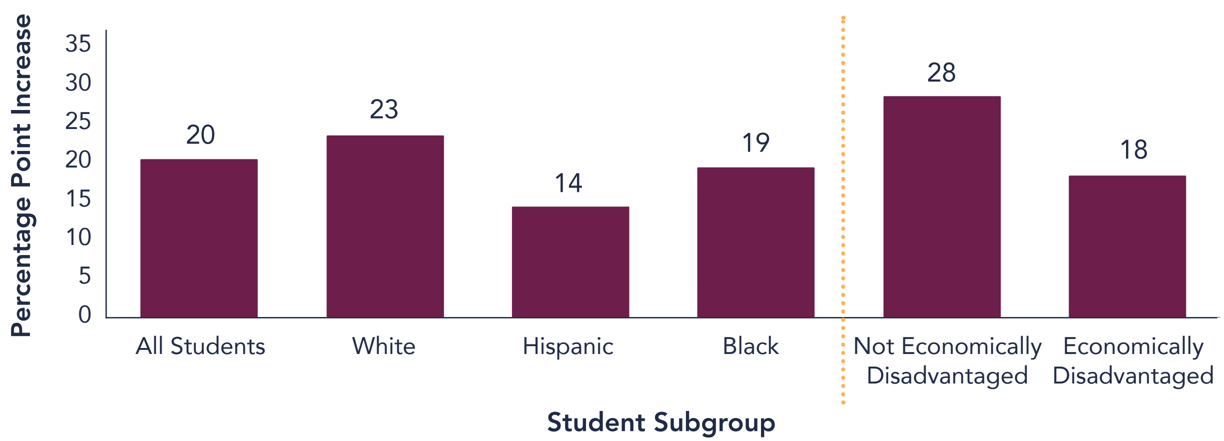
Rote Instruction and "Teaching to the Test": Many schools prioritized tested math content and skills, reducing time for exploratory projects, open-ended problem-solving, and deeper conceptual work that would not appear on state standardized tests. Critics argued that a fixation on boosting test scores often narrowed and "dumbed down" the curriculum and fostered a "drill-and-kill" instructional approach characterized by procedural repetition and heavy use of practice tests, especially in lower-performing schools facing the highest accountability pressure.

Some evidence indicates that NCLB had a positive effect on math performance and achievement gaps.

One widely cited causal study found that NCLB's standards and accountability pressures generated statistically significant increases in fourth-grade math achievement as well as smaller improvements in eighth-grade math. Gains were particularly strong among Black, Hispanic, and low-income students. Another study found that schools that received failing grades often saw learning gains in later years.

However, NCLB gains in math performance varied by school and district context. Schools and districts with strong leadership, rigorous instructional materials, expert teachers, and prior experience with standards-based reforms tended to make larger gains compared with schools and districts that lacked these things. Teacher knowledge and expertise in math continued to be an obstacle for many schools despite NCLB's "high-quality teacher" requirements.

Percentage Point Increase in NAEP Fourth-Grade Math Proficiency Rate, by Subgroup, 2000-2013



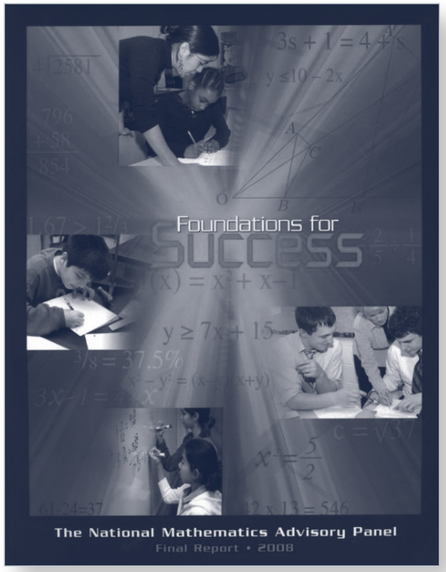
Between 2000 and 2013, fourth-grade math scores increased for all student subgroups. Racial and socioeconomic achievement gaps narrowed. Eighth-grade math scores also increased, but by a smaller amount. Gains were most concentrated around the proficiency threshold.

In 2006, the George W. Bush administration convened an expert National Mathematics Advisory Panel (NMAP).

NMAP included education researchers, mathematicians, cognitive scientists, educators, and other experts. **The panel was tasked with strengthening math policy and instruction in the U.S., particularly in the early grades leading up to algebra.** NMAP's 2008 report *Foundations for Success* included several key recommendations:

- Streamlined pre-K through Grade 8 curriculum anchored in the most critical skills needed on the path to algebra.
- Deeper engagement with high-quality research about how students learn math, particularly vis-à-vis instructional decision-making.
- Greater investments in mathematically knowledgeable teachers, including improvements in educator recruitment, preparation, retention, and evaluation.
- Improved state and national assessments of math performance.
- Increased funding for math education research.

NMAP struck a conciliatory tone in the math wars and advocated for a balanced approach to instruction, helping to reset the national conversation around focus, coherence, and engagement with scientific evidence about what works.



"Debates regarding the relative importance of conceptual knowledge, procedural skills … [and] memory are misguided. These capabilities are mutually supportive, each facilitating learning of the others." —NMAP, 2008

2010's CCSS aimed to improve state math standards' focus, coherence, and rigor.

CCSS emerged from a state-led initiative spearheaded by the National Governors Association and Council of Chief State School Officers. CCSS was framed as a response to uneven (and often weak) state academic standards and mediocre student performance on national and international assessments. The math standards (CCSSM) aimed to raise the bar for students and unify K-12 math expectations nationwide through three design shifts.

- **Focus**: A narrower curriculum in each grade level to combat the critique that U.S. math was a "mile-wide, inch-deep." Teachers would be expected to prioritize fewer, more essential topics at each grade level and deepen instruction in those topics.
- **Coherence**: A logical progression of math concepts and skill-building within and across grade levels, with each year's content learning building on prior knowledge.
- **Rigor**: A pursuit of conceptual understanding, procedural skills, and application with equal intensity so that students develop a deep and authentic command of math concepts.

CCSSM introduced eight research-backed standards for mathematical practice (habits of mind) that educators should inculcate in students, including problem-solving, reasoning, modeling, argumentation, and precision.

> *"For years there has been a raging debate ... procedural fluency or conceptual understanding. The obvious answer is both, and the standards give that answer."*
> —BILL MCCALLUM, FORMER PROFESSOR AND CCSSM LEAD DRAFTER

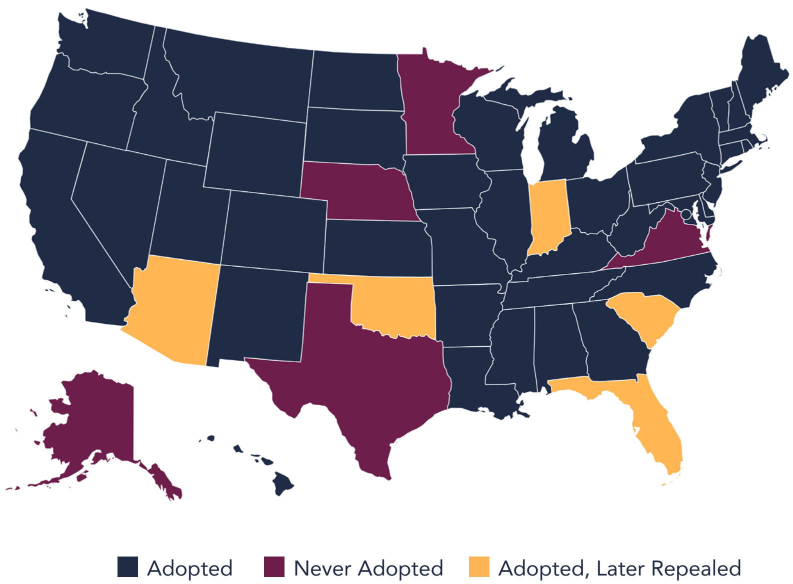
> *"Mathematics is not a list of disconnected topics, tricks, or mnemonics; it is a coherent body of knowledge made up of interconnected concepts."* —CCSS

Several implementation obstacles limited the ability of CCSS to transform math instruction.

- **Misaligned Curricula**: Securing HQIM aligned with the new standards was difficult, and teachers often used self-developed or subpar materials that undermined the goals of curricular rigor and coherence.
- **Inadequate Teacher Professional Development**: The standards implied shifts in instruction as teachers were expected to facilitate deeper discussions, use rich problems, and connect mathematical ideas across topics. The rushed rollout of CCSS in many states meant that professional development was often insufficient for teachers to fully understand and effectively teach the new standards, particularly vis-à-vis conceptual learning.
- **Political Polarization**: Critics on the political right attacked Common Core as a "national curriculum" imposed by the federal government, especially after the Obama administration incentivized states to adopt CCSS through the Race to the Top program. Critics on the political left objected that the higher standards would unfairly punish schools and educators for poor academic performance.
- **Entwinement With High-Stakes Testing and Teacher Evaluation**: CCSS was implemented at the same time as new state assessments and high-stakes teacher evaluation systems. When test scores dropped as a result of the higher standards and the switch to CCSS-aligned assessments, educators, teachers unions, and families protested that their local schools were being set up for failure.
- **Family Math Anxiety**: The CCSS rollout confused many parents, who saw their children struggling with new concepts and encountered unfamiliar methods on their homework. Many parents perceived "new math" as adding unnecessary complexity to simple procedures.

While 45 states and the District of Columbia adopted CCSSM, implementation challenges led to a retreat in support over time.

CCSSM Adoption Status, by State and the District of Columbia, 2021



A few early adopter states repealed the CCSS after initially adopting them while others made revisions to the standards. By the late 2010s, most states had revised or rebranded their standards, avoiding the Common Core name but keeping the content mostly unchanged. As a result, in 2020 most states had math standards very similar to the original CCSSM.

Percent of Adults Who Support CCSS, 2012-2016

Public support for the Common Core declined dramatically between 2012 and 2016, with the new math standards standing out as a particular source of discontent. A 2015 PDK/Gallup poll found that 60% of Americans opposed the Common Core, partly due to concerns about teacher opposition to CCSS, limited teacher flexibility in delivering instruction, and federal involvement in CCSS.

CCSSM's most significant effects were on curriculum and instructional practices; effects on learning appear more mixed.

INSTRUCTION EFFECTS

- Teachers reported increases in teaching grade-level topics delineated in CCSSM, focusing more on math applications, spending less time on rote memorization, and teaching multiple methods to solve problems.
- Teachers reported using CCSS-aligned textbooks in their classrooms and reported that alignment to CCSS was one of the three most important influences on curricula, programs, and/or instructional tools used in their classroom practices.

The Obama administration's Race to the Top program incentivized states to adopt CCSS as they competed for grant dollars. One study found that students in states that applied for or won this program's grants demonstrated greater math gains compared with states that did not.

ACADEMIC OUTCOMES EFFECTS

- One cross-state study found that early adoption of CCSS was associated with small, positive effects in math performance, while another study in Kentucky found that students exposed to CCSS earlier during implementation made faster learning progress.
- Another cross-state study found that states that changed their standards most dramatically in response to CCSS experienced modest and consistent declines in academic performance, especially in eighth-grade mathematics.

CCSS's biggest downstream effect was catalyzing a national market for standards-aligned HQIM, transforming curriculum publishing and procurement and making instructional materials a central lever for states and districts trying to improve math instruction.

The 2015 Every Student Succeeds Act (ESSA) gave states more autonomy over math standards and accountability.

ESSA preserved federal requirements for annual testing and performance reporting. States continued to be required to test all students in Grades 3-8 and once in high school annually and publish school report cards showing performance disaggregated by student subgroups.

ESSA also granted states more authority to design accountability systems and intervention strategies. States were required to include four academic factors in their accountability policies: reading and math test scores, English-language proficiency rates, high school graduation rates, and another state-chosen academic measure such as student growth. They also were required to consider an additional "fifth indicator" of school quality, such as school climate, college readiness, absenteeism, or access to advanced coursework. States were responsible for setting their own achievement targets and intervention strategies for low-performing schools.

ESSA prohibited the federal government from promoting or prescribing a particular set of standards in response to previous federal incentives to adopt CCSS.

Since the enactment of ESSA, math performance in the U.S. has declined — leading some observers to claim that the legislation allowed states to "water down" standards and accountability. However, there is little causal evidence linking ESSA to performance trends, and most studies of ESSA-era accountability largely find mixed or null effects on student outcomes. Studies from Ohio and California, for example, showed that schools targeted for improvement under ESSA showed no significant gains in academic outcomes — perhaps because school-level interventions to improve performance were not always linked to coherent turnaround strategies and research-based evidence or because the additional resources and support provided to these schools were inadequate.

ESSA's main influence on math is seen in assessments, teacher professional development, school quality measurement, and evidence use.

State Assessments

Advanced eighth-graders who are taking a high school math course can sit for the aligned end-of-course exam instead of the usual grade-level test. This reduced double-testing and enabled states to experiment more with accelerated math pathways. ESSA allows states to use a nationally recognized assessment (e.g., SAT, ACT) for their high school state exam. Nineteen states require the SAT or ACT for their high school math assessment.

Teacher Professional Development

ESSA eliminated the federal "Highly Qualified Teacher" requirements and emphasized capacity building instead. The law channeled resources into developing educators, such as Title II funds that supported new structured pathways for training teachers like the STEM master teacher corps to improve math instruction.

School Quality Measurement

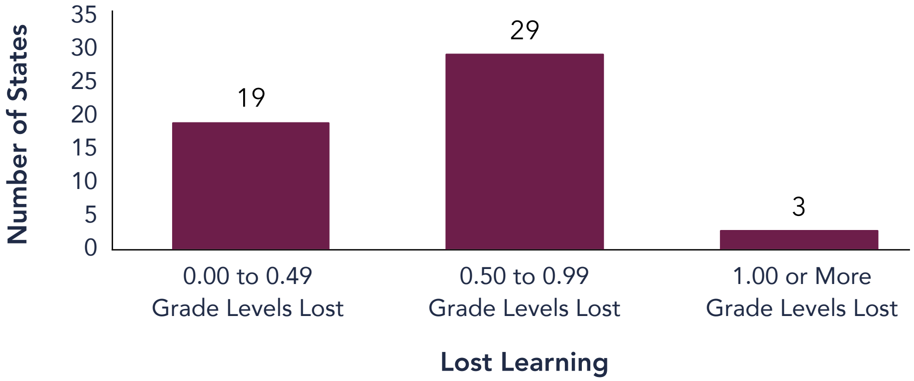
Math is an input in many states' fifth indicator school quality measures, particularly in high school. These often take the form of college- and career-readiness measures that include math benchmarks, math course-taking or completion data, and achievement composites that include math performance. Some states' ESSA school report cards provide robust and easy-to-access information about math performance and opportunity, but many other states lag in this area.

Evidence Use

ESSA requires many federal funds to be spent on "evidence-based" curricula and interventions, defining clear tiers of evidence and nudging states and districts toward rigorously evaluated programs with demonstrated impact.Resources like the What Works Clearinghouse, Evidence for ESSA, and EdReports have helped leaders identify vetted math programs and instructional resources. Many school improvement plans prioritize research-based math strategies (e.g., coaching, targeted professional development, high-dosage tutoring) to ensure funds are used for what works.

The COVID-19 pandemic sharply intensified math performance declines and spurred acceleration efforts.

Math Learning Loss Across States and the District of Columbia, 2019-2022



The average student in Grades 3-8 lost more than half a year of learning in math (-0.53 grade levels) between 2019 and 2022. Every state and the District of Columbia experienced learning loss, ranging from Alabama (-0.12 grade levels) to Delaware (-1.25 grade levels).

Between 2020 and 2024, states spent billions of federal education pandemic relief dollars on math interventions such as tutoring, summer and after-school learning, and HQIM to support academic recovery and learning acceleration. Research indicates that between 2022 and 2024, students recovered about one-fifth of their math learning loss (~0.10 grade levels). States and districts that spent more of their funds on direct academic recovery supports saw the largest student learning gains. While some districts have recovered math performance to pre-pandemic levels, many remain below that level. Nationwide, higher-income districts are approximately four times more likely to have recovered. Alabama is the only state to have fully recovered in math.

Notable Investments

- **Louisiana** scaled access and supports for Zearn, a digital tool to help students catch up in math, and saw gains for frequent-user students (Disclosure).
- **Illinois'** statewide tutoring initiative drove math learning gains, especially among high-need students.
- **Tennessee's** ALL Corps tutoring program boosted math achievement for students.
- In **Texas**, Ector County Independent School District invested in tutoring and a redesign of teacher roles to boost achievement.
- In **California**, Compton Unified School District invested in Saturday and summer school programs and data-driven decision-making professional development for educators.

The racial justice movement sparked in 2020 led to new efforts to democratize access to math content. (1/2)

A push to detrack math courses and address the disproportionately low rates of Black, Latino, and low-income students in higher-level math courses accelerated after 2020 amid national discussions about racial and economic justice following the murder of George Floyd.

These efforts — in which all students were placed in similar, mixed-ability math classes — were often highly controversial, pitting arguments for educational equity and social justice against arguments for traditional standards and support for advanced students.

Districts that led detracking initiatives during this time included Massachusetts' Cambridge Public Schools, Michigan's Troy School District, New York's Ithaca City School District, and California's San Francisco Unified School District — although the latter rolled back its decade-long effort in 2024. Other districts, like Union Public Schools (OK), sought a "middle ground" by offering more on-ramps to advanced courses (such as teacher referrals or repeated placement tests). Virginia and California debated statewide detracking initiatives before walking them back.

Automatic enrollment policies, in which students who meet objective criteria are automatically placed in advanced math classes, also expanded during this time. These efforts produced gains in the number of historically marginalized students enrolling in and sticking with advanced math courses. **Today, at least 11 states have automatic enrollment policies in effect.**

The racial justice movement sparked in 2020 led to new efforts to democratize access to math content. (2/2)

The focus on racial justice after 2020 also spurred discussions nationwide about embedding social justice into math courses and making instructional content more socially relevant and engaging for children — something that had been a focus of math reform advocates for several years.

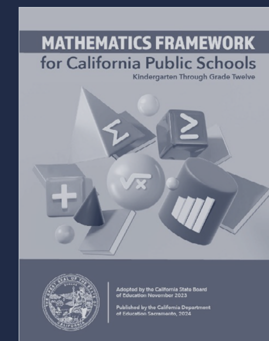
Although enacted legislative change was very limited, efforts to improve math equity and social relevance became elevated features of state and national policy discussions during this time.

These conversations did translate into localized shifts in policy and practice in some communities. Seattle Public Schools, for example, built on earlier launched efforts to "rehumanize" math through a social-justice-oriented K-12 Math Ethnic Studies Framework, and Georgia's Clayton County Public Schools developed its own social justice approach to math instruction across grades. Media reports described how teachers in various districts increasingly sought out math problems grounded in real-world data on policing, public health, and economic inequality, and a 2020 book of social-justice-infused math lessons authored by a former NCTM president became a best seller.

These initiatives drew significant public attention and criticism from some observers, who argued that such approaches represented a politicized form of "woke math."

In 2021, California proposed a controversial new math framework that reflected the major debates in this period.

The proposed framework focused on stringent detracking, delaying Algebra I until ninth grade, and de-emphasizing calculus as a math capstone. It also cautioned educators against labeling students as "gifted" and espoused an explicit social justice orientation to math teaching. The framework became an immediate flash point in national math wars. Proponents argued that the framework would enhance math access, learning, and engagement among historically marginalized students. Opponents raised serious questions about the quality and use of research to justify the reforms. Opponents also claimed it would "dumb down" instruction, hold back advanced students, underprepare students for STEM careers, and inject "woke ideology" into the classroom. A revised framework was finally approved in 2023 and walked back the most sweeping changes.



CORE THEMES IN THE FINAL APPROVED FRAMEWORK

- **Cross-Grade Concepts**: A shift from approaching math as a checklist of individual standards toward teaching through "big ideas" that integrate concepts across grade levels.
- **Inquiry-Based Learning**: An encouragement for teachers to embrace inquiry-based approaches and integrate real-world problem-solving to improve relevance for students.
- **Cultural Responsiveness**: An emphasis on making math class welcoming and empowering for students from all backgrounds by using culturally relevant teaching methods and examples that reflect students' lived experiences.
- **High School Course Sequencing**: A recommendation that most students take Algebra I in ninth grade, explicit acknowledgment that some students should be able to take it earlier, and suggestions to ensure accelerated learning opportunities are unbiased and available to more students.
- **Data Science Integration**: An encouragement to teach data science throughout all math pathways even while omitting a data science pathway as an alternative to Algebra II.

Many states and the District of Columbia have implemented policies to better align math pathways with college and careers.

Several states have restructured high school math course sequences and pathways to move beyond the traditional algebra-to-calculus track and better reflect students' diverse college and career goals. These policies aim to maintain rigor while expanding options — such as statistics, data science, and quantitative reasoning — to make advanced math more relevant and accessible to all learners. In recent years:

- **31 states** have formalized government bodies, task forces, or working groups to lead mathematics pathways reforms.
- **23 states** have taken steps to better align requirements for high school graduation with college admissions.
- **19 states** have implemented four-credit math requirements in high school to ensure that students take math continuously during those years.
- **29 states** have created policies or offered guidance around corequisite policies, which allow underprepared postsecondary students to be placed directly into credit-bearing, program-appropriate gateway math courses rather than remedial courses.

In many cases, the link between changes in policy and improved classroom practices is unclear and understudied.

ECE math has received less policy attention, and it is often subsumed under wider efforts to improve ECE quality.

In recent years, ECE policy has prioritized critical areas such as access, affordability, workforce pipelines, program quality, and K-12 alignment. Efforts to boost ECE academic outcomes have focused predominantly on literacy.

While rarely a central focus of ECE policy in itself, math is sometimes folded into broader ECE improvement agendas. Recent federal and state moves have helped normalize math in early-learning standards, expanded its presence in curricula and kindergarten readiness assessments, and encouraged stronger preschool to Grade 3 alignment. However, math continues to lag in ECE policy attention and investment.

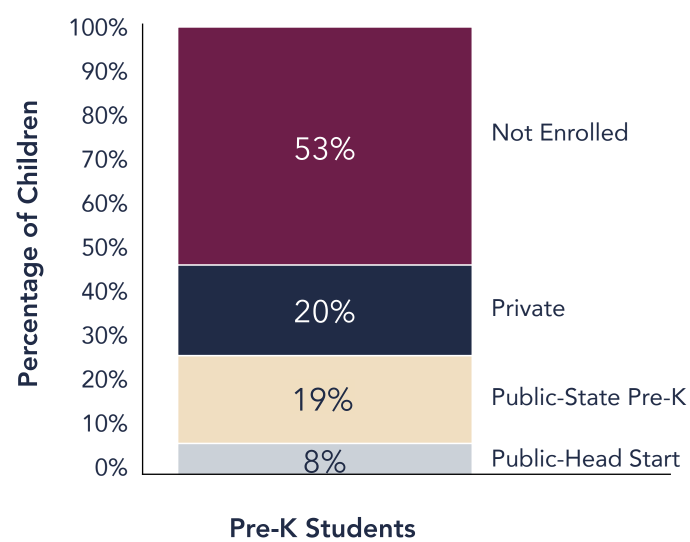
> *"The relative lack of high-quality mathematics instruction, especially in comparison to literacy, reflects a lack of attention to mathematics throughout the childhood education system, including standards, curriculum, instruction, and the preparation and training of the teaching workforce."* —NRC, 2009

Although there is no national assessment of ECE math performance, the data that does exist portrays stagnating or declining performance in recent years. National data from NWEA's Measures of Academic Progress (MAP) Test indicates that the school-entry math performance of kindergarteners declined modestly between 2010 and 2017 and has been stagnant since then. Data from Curriculum Associates' i-Ready exams reveals a decline in the percentage of kindergarteners scoring "on grade level" in math from 84% in 2019 to 70% in 2025.

Several barriers hamper policy efforts to improve ECE math performance.

- **Governance Complexity**: The mixed-delivery ECE landscape makes it difficult to set and monitor consistent math expectations. Fragmented authority within and across federal and state agencies also complicates reform efforts.
- **Blinkered Quality Assurance Systems**: Preschool licensing and monitoring systems (i.e., Quality Rating and Improvement Systems) emphasize structural factors and process variables but typically lack tools that directly observe and rate math instruction.
- **Inadequate Educator Preparation and In-Service Training**: State credentialing guidelines and preparation programs often devote very little attention to early math, leaving new teachers underprepared. On-the-job support is also limited.
- **Uneven Standards, Assessments, and Curricula**: State ECE math standards often emphasize rote skills over deep understanding, curricula can be weak, and expectations are sometimes misaligned between preschool and kindergarten, prompting unnecessary repetition. States' assessment systems rarely share preschool data with K-12 schools.
- **Missing Guidance for Families and Educators**: Many families — and some educators — view mathematics as "not for preschool" or worry that it crowds out play, leading to lower expectations and limited math instruction in the classroom and at home. Programs and families often lack clear guidance for everyday math routines.

Pre-K Enrollment of 3- and 4-Year-Olds, 2022



> *"Aligning bad instruction in preschool with bad instruction in early elementary is obviously not desirable. In addition to expanding attention to math in preschool and the early grades, states need to improve instruction."* —DEBORAH STIPEK, PROFESSOR, STANFORD UNIVERSITY

Although attention for early math lags literacy, several federal initiatives have advanced ECE math since 2000.

- **Good Start, Grow Smart Initiative (2002)**: Incentivized states to create voluntary early-learning guidelines aligned to K-12 and required Head Start centers to assess early literacy and numeracy standards.

- **Improving Head Start for School Readiness Act (2007)**: Reauthorized Head Start and strengthened expectations for academic development in multiple domains, including math. In subsequent years, the Office of Head Start released a revised Early Learning Outcomes Framework (ELOF), which laid out clear expectations for preschoolers' math development, offered new technical assistance focused on early math skills, and introduced the Head Start Designation Renewal System, which tied funding to indicators of school readiness, including math skills.

- **Race to the Top — Early Learning Challenge (2011-2013)**: Awarded competitive grant funds to states to improve their early childhood systems. These grants incentivized states to develop comprehensive, high-quality early learning standards and implement kindergarten entry assessments in multiple domains including math. Some winning states invested in math or STEM-focused professional development for ECE educators and strengthened quality rating and improvement systems.

- **Preschool Development Grants (2014-2018)**: Helped states expand high-quality pre-K for 4-year-olds. These grants expected states to use evidence-based curricula and measure child outcomes in math and literacy to monitor progress.

In 2009, a federally funded NRC report sparked renewed policy conversations about how to improve ECE math.

The report *Mathematics Learning in Early Childhood: Paths Toward Excellence and Quality* offered an urgent call to improve ECE math instruction and offered several major recommendations:

- Establishment of a coordinated national early childhood mathematics initiative to improve math teaching and learning for children ages 3-6.
- Implementation of high-quality math curricula and instruction in ECE settings.
- State-level development or revision of ECE math learning standards.
- Strengthened professional development for in-service ECE teachers to deepen understanding of math content, pedagogy, and curriculum implementation.
- Greater focus on ECE math in educator preparation coursework and practicum requirements.
- Stronger curricular focus on numbers (e.g., whole numbers, operations, and relations) as well as geometry, spatial relations, and measurement in ECE settings.

The report was a catalyst for various professional associations, foundations, and state governments to update their ECE math guidance, increase funding for early math initiatives, and improve instruction. It also laid the groundwork for later conversations about strengthening pre-K through Grade 3 alignment in math.

> *"Many early childhood settings do not provide adequate learning experiences in mathematics. ... When early childhood classrooms do have mathematics activities, they are often presented as part of an integrated or embedded curriculum, in which the teaching of mathematics is secondary to other learning goals."* —NRC, 2009

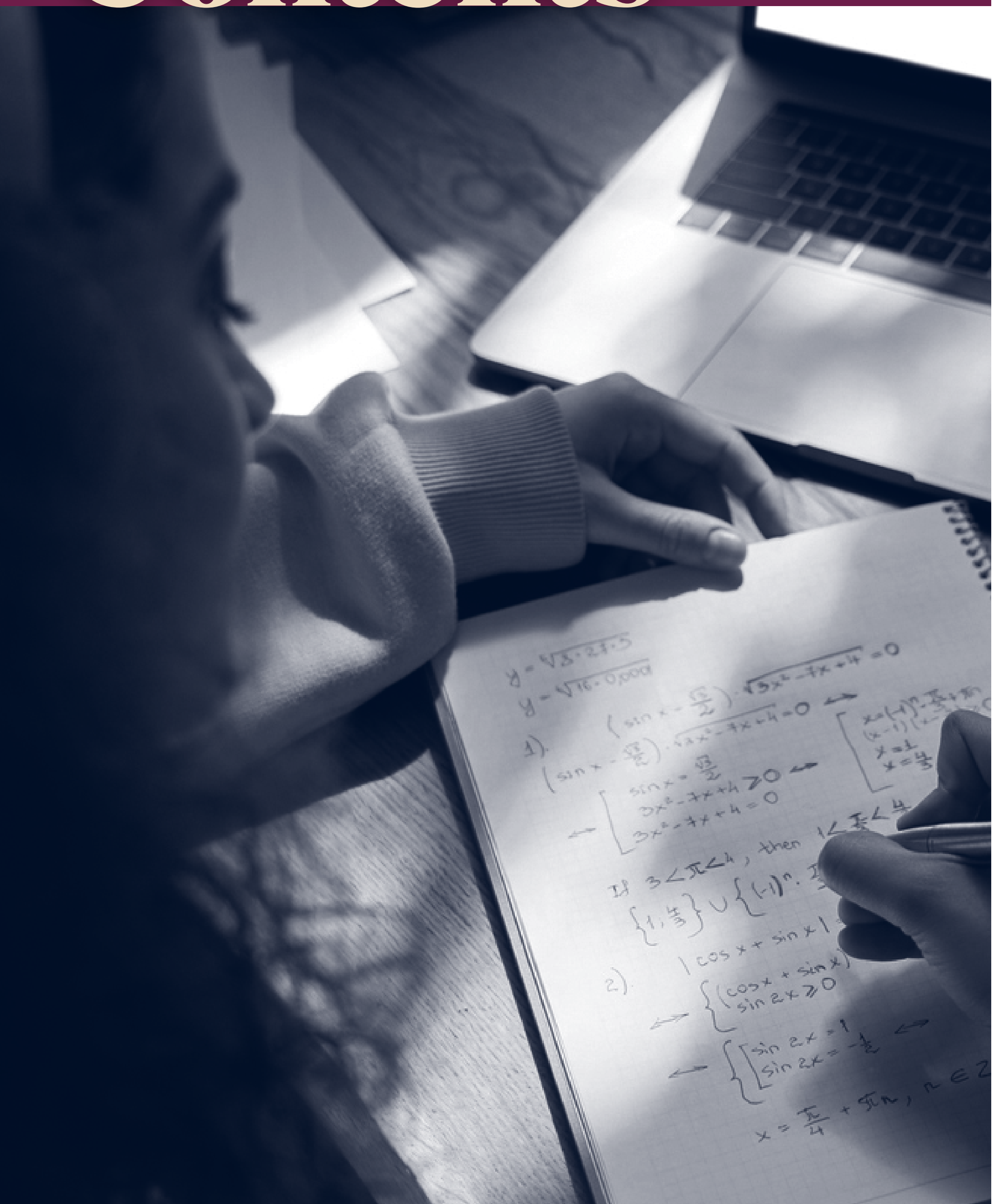
A number of states and school districts have tried to strengthen ECE math instruction in recent years.

In 2018, the **California Early Math Initiative** was launched to develop and scale innovative professional learning, coaching, and resources to build the math content knowledge and leadership capabilities of educators and families serving children ages 0-8. An evaluation found that participating educators reported increases in math content knowledge, confidence, and teaching practices. In Fresno County, the initiative helped deliver coaching-rich professional learning and shift instructional practice toward play-based, conceptual teaching. In 2025, California announced new pre-K through Grade 3 learning progressions that aim to strengthen cohesion in early math teaching and learning. The learning progressions built on successful pilots of P-3 coherence in a handful of districts for those grade levels.

In 2012, **New York City** launched the Making Pre-K Count initiative, a public-private partnership with MDRC and the Robin Hood Foundation. The initiative provided an enhanced math curriculum and coaching to dozens of community pre-K centers and was intentionally designed to evaluate the impact of early math interventions on children's learning and later life outcomes.

The **Texas Education Agency** released *Texas Prekindergarten Guidelines* in 2022 that provide guidance on quality learning experiences for children ages 3-5. The guidelines include mathematics content knowledge and skills and are aligned with the state's K-12 academic standards.

Contents



Persistently low math performance has spurred several states to pursue innovative math policies in recent years.

Despite pockets of growth over the past decade (particularly in the South), math performance remains low across all 50 states and the District of Columbia. This is true even among the highest-performing states.

NAEP Fourth-Grade Math Proficiency Rates, 2024

Top Five States	Math Proficiency Rate
Massachusetts	51%
Wyoming	46%
Utah	45%
Florida	45%
Minnesota	45%
U.S. Average	**39%**

Highest-Growth States Since 2013

Mississippi (+12), Louisiana (+8), Alabama (+7)

NAEP Eighth-Grade Math Proficiency Rates, 2024

Top Five States	Math Proficiency Rate
Massachusetts	37%
New Jersey	37%
Wisconsin	37%
Utah	35%
Minnesota	34%
U.S. Average	**27%**

Highest-Growth States Since 2013

Tennessee (+3), District of Columbia (+1)

In the wake of stalled performance, some states have developed policies to improve math learning, access, and relevance. These policies have targeted all levels of math, from pre-kindergarten to K-12 to postsecondary education. Three noteworthy state initiatives are found in **Maryland**, **Ohio**, and **Alabama**. Each of these state initiatives is in a different stage of implementation, and it is still too early to know their full effects on classroom practice, but they have the potential to serve as models for other states.

States to Watch: Maryland's 2025 Pre-K Through Grade 12 Math Policy (1/4)

Maryland is facing a persistent math achievement crisis following a decade of declining performance and wide achievement gaps.

In 2024, Maryland ranked 35th and 32nd on the NAEP fourth- and eighth-grade math exams, respectively. In Baltimore, only 9% of eighth-grade students scored proficient on the NAEP math exam. Results from state standardized tests are hardly better, with just 24% of students statewide scoring proficient on the Maryland Comprehensive Assessment Program math tests in the 2023-24 school year (SY).

In summer 2025, Maryland adopted a pre-K through Grade 12 math policy reform plan that tackles everything from classroom instruction and course sequences to standards, curriculum, tracking, and professional development at all levels of the system. It reflects a similarly broad-ranging approach to instructional improvement seen in "Science of Reading" literacy reforms.

> Three bright spots in this reform involve new requirements for early math, an inclusive approach to instructional grouping, and a revamped secondary math sequence better connected to college and career readiness.

States to Watch: Maryland's 2025 Pre-K Through Grade 12 Math Policy (2/4)

STRONGER EXPECTATIONS FOR EARLY MATH (PRE-K THROUGH GRADE 5)

- Revisions to pre-K and kindergarten standards will clarify expectations and strategies to strengthen foundational numeracy skills as well as integrate early concepts needed for geometric and algebraic reasoning.
- Students in K-8 will be required to have 60 minutes of math instruction every day, up from 45 minutes in most districts.
- Districts will be required to establish a process for identifying and supporting students who struggle with math in alignment with a statewide numeracy development framework.

Under the new standards, PreK students will, among other skills, be expected to count to 20, recognize numerals, count backwards, work in repeating patterns, and develop geometric reasoning by describing and comparing basic shapes.

INCLUSIVE APPROACH TO INSTRUCTIONAL GROUPING

- School districts are required to reduce or eliminate exclusionary tracking in mathematics education and ensure students are purposefully grouped in classrooms where all students have access to effective mathematics instruction.
    - In pre-K through Grade 1, schools are required to maintain heterogeneous mathematics classrooms, leveraging flexible grouping to support and enrich student learning.
    - In Grades 2-5, schools are expected to "purposefully and regularly regroup students for math instruction based on [local education agency] LEA developed [Multi-Tiered Systems of Support] MTSS math structures," including acceleration opportunities and small group instruction.
- All students entering Grades 3-7 are required to be evaluated for math acceleration readiness at least once a school year.

States to Watch: Maryland's 2025 Pre-K Through Grade 12 Math Policy (3/4)

TRANSFORMATION OF THE SECONDARY COURSE SEQUENCE

- Maryland schools will transition away from the traditional Algebra I-Geometry-Algebra II sequence and replace it with a rigorous, two-year integrated algebra sequence that blends algebra, geometry, and data analysis.
- The goal of the new sequence is to build a stronger foundation for advanced math and, by compressing the core high school math into two years, expand options for what students can do in 11th and 12th grade.
- Later coursework will reflect revamped math pathways better aligned with different college and career aspirations.
  - **Quantitative Reasoning**: Courses develop real-world mathematical skills in problem-solving, modeling, financial literacy, and data-driven decision-making.
  - **Data and Data Analytics**: Courses build skills in data analysis, programming, and mathematical reasoning. They leverage technology to explore real-world datasets and prepare students for a data-driven future.
  - **Algebraic Foundations of Calculus**: Courses are designed for students pursuing STEM fields and provide a deep exploration of functions and change in preparation for calculus.
  - **Statistics and Probability**: Courses build students' understanding of data, uncertainty, and statistical inference.

> Maryland's overhaul of its secondary course sequence grew out of a multiyear engagement with the Charles A. Dana Center's Launch Years Initiative, which supports dozens of states in modernizing secondary math course sequences and developing open, relevant pathways (Page 65).

States to Watch: Maryland's 2025 Pre-K Through Grade 12 Math Policy (4/4)

Maryland's math policy implementation plan contains several strengths:

- **Integration With HQIM** — The Maryland State Department of Education's (MSDE's) HQIM Branch will review the most commonly used curricula in the state to, among other things, ensure alignment with the new standards and facilitate Common Curriculum Communities with LEA Mathematics Supervisors to prepare resources, strategies, and professional learning supports for educators making curricular and instructional shifts.
- **Investments in Professional Learning** — Local LEA teams will serve as liaisons to support shifts in math standards professional learning and collaborate closely with MSDE in strengthening educator capacity for mathematics instruction.
- **A Focus on Building Math Identity** — There is an explicit focus on strengthening students' attitudes toward math and building positive math identities. Educators are expected to provide experiences that help students see themselves as capable mathematicians who experience a sense of belonging in their mathematics classroom.

States to Watch: Ohio's Strengthening Ohio High School Mathematics Pathways Initiative (1/3)

Launched in its first classrooms during SY22-23, Ohio's Strengthening Ohio High School Mathematics Pathways Initiative aimed to address three main problems:

- **High School Graduation Hurdles** — Ohio students must earn credit in Algebra II (or an equivalent course) to graduate, but the course had become a barrier for many students. Of all Ohio students who did not graduate, over 94% had not earned credit in Algebra II or an equivalent course.
- **Misalignment With the State's Strategic Education Plan and Workforce Needs** — Ohio's strategic education plan ("Each Child, Our Future") emphasized multiple pathways to success, but the state had essentially only one mathematics pathway for students.
- **Misalignment With Postsecondary Expectations** — Beginning in 2013, the state's public colleges and universities had redesigned entry-level college math offerings through the Ohio Mathematics Initiative, aiming to better align with different fields of study and replace the old "college algebra for all" approach. This created misalignment between postsecondary expectations and algebra-focused high school math curricula.

On state standardized tests in SY24-25...

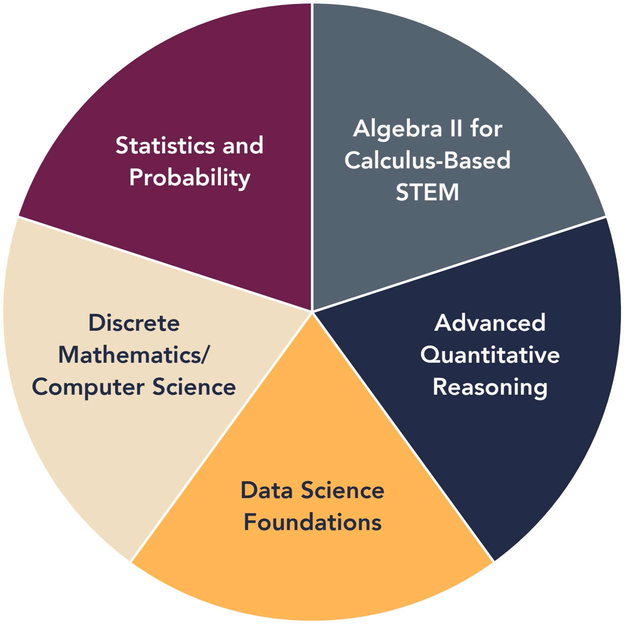
59% of Ohio students scored proficient in Algebra I

63% of Ohio students scored proficient in geometry

States to Watch: Ohio's Strengthening Ohio High School Mathematics Pathways Initiative (2/3)

The initiative created **five new course options** that local education agencies can implement to satisfy the Algebra II graduation requirement. All pathways are designed to be **rigorous**, **relevant**, **coherent**, and **flexible** while covering different content to prepare students for diverse career fields. As of 2025, nearly 30% of Ohio high schools offer at least one of the alternative math pathways to students.

Statistical inference, probabilistic reasoning, data interpretation, and real-world application. Targeted for students pursuing statistics-oriented fields like **social sciences**, **health care**, or **research**.



Algebraic reasoning and advanced functions. Targeted for students pursuing **calculus-based STEM fields**.

Introduction to topics like logic, algorithms, graph theory, and combinatorics that underlie computer science. Targeted for students pursuing fields such as **computer science and information technology**.

Application of algebra, geometry, and statistics to real-world problems to build critical thinking and decision-making skills. Targeted for students pursuing **non-STEM or applied fields such as education, the arts, or trades**.

The use of data analysis tools, algorithms, and machine learning to build skills in computational thinking, quantitative reasoning, and insight generation. Targeted for students pursuing **data-driven careers in fields such as business, health care, or analytics**.

States to Watch: Ohio's Strengthening Ohio High School Mathematics Pathways Initiative (3/3)

Three notable strengths of Ohio's math policy are coordination and alignment, implementation support, and family and student engagement.

- **Coordination and Alignment Among High School, Postsecondary, and Workforce Systems** — The Ohio Department of Education and Workforce partnered closely with the Ohio Department of Higher Education and representatives of the Ohio Mathematics Initiative to design the pathways. A working group of college and high school math faculty dubbed the "Math Pathways Architects" helped align high school course content with entry-level college math courses. An advisory council included stakeholders such as Ohio Excels (a business-education advocacy organization), Ohio Association for Career and Technical Education, and the Ohio Math and Science Coalition to facilitate workforce coordination.

- **Implementation Support for Schools, Colleges, and Educators** — The state provided extensive guidance to schools and colleges on how to implement math pathways, including a toolkit for high school math pathways and resources for connecting them to Ohio's College Credit Plus (dual enrollment) courses. In addition, the state has provided for extensive professional development for educators. This includes a structured two-year professional development program for educators, the preparation of "regional facilitators" to serve as trainers and mentors for new cohorts of teachers, and curricular resources and frameworks for school districts implementing the new pathways.

- **Family and Student Engagement** — The Ohio Department of Education and Workforce provides resources to families and students to help assess which math pathway is best for them, including course descriptions, decision trees, and resources about postsecondary credit options. The Department also hosted a recorded high school math pathways symposium in 2021 attended by hundreds of participants. Recorded sessions provided key details and guidance about the initiative, the new courses, the role of HQIM, and the implications of the policy for students, parents, school administrators, and higher education admissions.

States to Watch: Alabama's Numeracy Act of 2022 (1/3)

For years, Alabama had struggled with very low and inequitable math performance. On the 2019 NAEP exam, Alabama fourth graders ranked 52nd in math (ahead of only Puerto Rico), with a statewide proficiency rate of 28%. Results on the Alabama Comprehensive Assessment Program (ACAP) state tests were just as poor. In 2021, only 22% of students scored proficient on ACAP, including just 7% of Black students and 11% of students from low-income families. More than 70% of Alabama districts had proficiency rates below 25%, and 28 K-5 schools had proficiency rates of 0%.

To address this crisis, in 2022 the state legislature passed a comprehensive math policy called the Alabama Numeracy Act whose core aim is to ensure K-5 students are on grade level in math by the end of fifth grade. The law creates structures to identify schools in need of support, then deploys regional teams, K-5 math coaches, high-quality curricula, MTSS interventions, targeted summer math programs, and training for teachers and principals to improve performance. Additionally, the law promotes early student screening for math challenges, and it creates a statewide task force to recommend high-quality curricula, professional development, and assessments.

States to Watch: Alabama's Numeracy Act of 2022 (2/3)

The law established the Office of Mathematics Improvement with several duties.

IDENTIFICATION	SUPPORT	ACCOUNTABILITY	EDUCATOR PREPARATION
Identify low-performing elementary schools for intensive math supports, including the bottom 5% of schools (targeted for "full support") and bottom 25% (targeted for "limited support"). Oversee the implementation of K-5 screeners and diagnostics to track student progress and identify students with math learning challenges.	Vet, approve, and support the implementation of HQIM in math. Support the implementation of high-quality interventions for struggling students and schools, including summer programs. Manage the placement of math coaches in every K-5 school. Provide a continuum of professional development training for math educators.	Oversee locally led and state-supported turnaround strategies for schools not making math progress. After three years of full support and four years of intensive turnaround support without improvement, local boards must choose an intensive turnaround strategy like reconstitution or conversion to a charter school.	Convene and oversee a Postsecondary Mathematics Task Force to develop research-based guidelines for training and preparing ECE and elementary math teachers. Support the implementation of Alabama's instructional leadership framework for K-5 administrators and the state's School Turnaround Academy.

States to Watch: Alabama's Numeracy Act of 2022 (3/3)

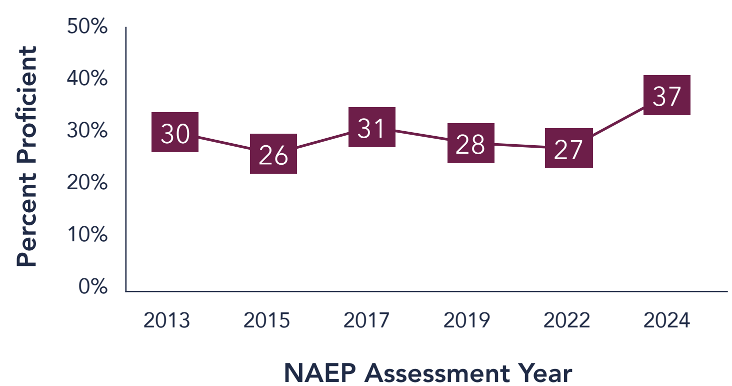
EVALUATION DATA REVEAL SEVERAL EARLY INDICATORS OF IMPLEMENTATION SUCCESSES

- **Buy-In**: Stakeholders across the system, including LEA staff, principals, teachers, and math coaches, report that they understand their new responsibilities under the Numeracy Act, have received necessary training and professional development, and have access to needed resources and supports.
- **Instructional Shifts**: More than 80% of K-5 teachers report spending the required 60 minutes daily on math instruction, using only high-quality curricula approved by the state task force, applying evidence-based teaching practices, and building students' conceptual understanding, strategic reasoning, and problem-solving.
- **Data Use**: Educators report using data more frequently to assess student progress and communicate it with families. Screeners are helping teachers pinpoint students' specific strengths and difficulties for targeted interventions.

TEST SCORES IN ALABAMA HAVE IMPROVED DRAMATICALLY IN RECENT YEARS

- **Alabama ranks 1st among states in pandemic math recovery,** having gained 0.09 grade levels in math learning since 2019 (compared with the national average of -0.46 grade levels). Alabama is the only state to have fully recovered in math.
- **Alabama leads the nation in fourth-grade math NAEP proficiency gains since 2019.** It has jumped from being ranked 52nd in math proficiency to 35th.*

Alabama NAEP Fourth-Grade Math Proficiency Rate, 2013-2024



Notes: *Calculations based on all public schools in Alabama. A 2025 press release from the Alabama Department of Education stated that the state had improved from 52nd to 32nd.

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Foundations in quality math policy and practice exist, but implementation is a barrier to improved outcomes.

Math skills are critically important for students' future academic and socioeconomic outcomes, but performance on math assessments is distressingly low.

Although math does not have a clear analog to the Science of Reading, there is a solid and growing evidence base about what math education can look like for students in different stages of their academic careers, including effective instructional approaches, standard sequences of content and learning progressions, and strategies for targeting instruction to student needs and abilities.

Numerous policy reforms have taken place over decades and have shaped key elements of math education, including academic standards and content, assessment programs, instructional practices, course access, social relevance, and student pathways. However, efforts at the national level have left gaps in policy leadership and implementation at the state and local levels. Compared with K-12 math and ECE reading, ECE math has been underemphasized in policy circles.

Efforts to reform math policy and practice often confront obstacles in the form of inadequate teacher preparation, uneven implementation of HQIM and aligned professional development, missing or incoherent state policies, and a culture of math anxiety. States that are leading in math policy today are taking steps to address many of these barriers.

Promising signs are emerging for improving math instruction in the field.

As public conversation about improving math policy and practice continues to grow, policymakers, practitioners, and other stakeholders can:

Continue to use and strengthen research to deepen understanding of how and when young people most effectively learn math, including identifying the most appropriate and high-impact instructional approaches, content standards, and course sequences across the pre-K through Grade 12 continuum.

Consider math as a cumulative, cradle-to-career progression rather than a series of disconnected courses.

Build on efforts that help ensure all students have access to high-quality math instruction, such as investments in improved academic standards, accountability systems, educator preparation and in-service support, instructional materials, family engagement, and math pathways aligned with postsecondary options.

Learn from states and districts that are currently investing in improving math education by setting reform strategies and leveraging research and data in their efforts.

> Progress in math education will require concerted effort across research, policy, and practice and an honest commitment to grounding in facts, evidence, and data. Those in positions to influence improvements in this space must understand it, and this stocktaking report supports that goal.

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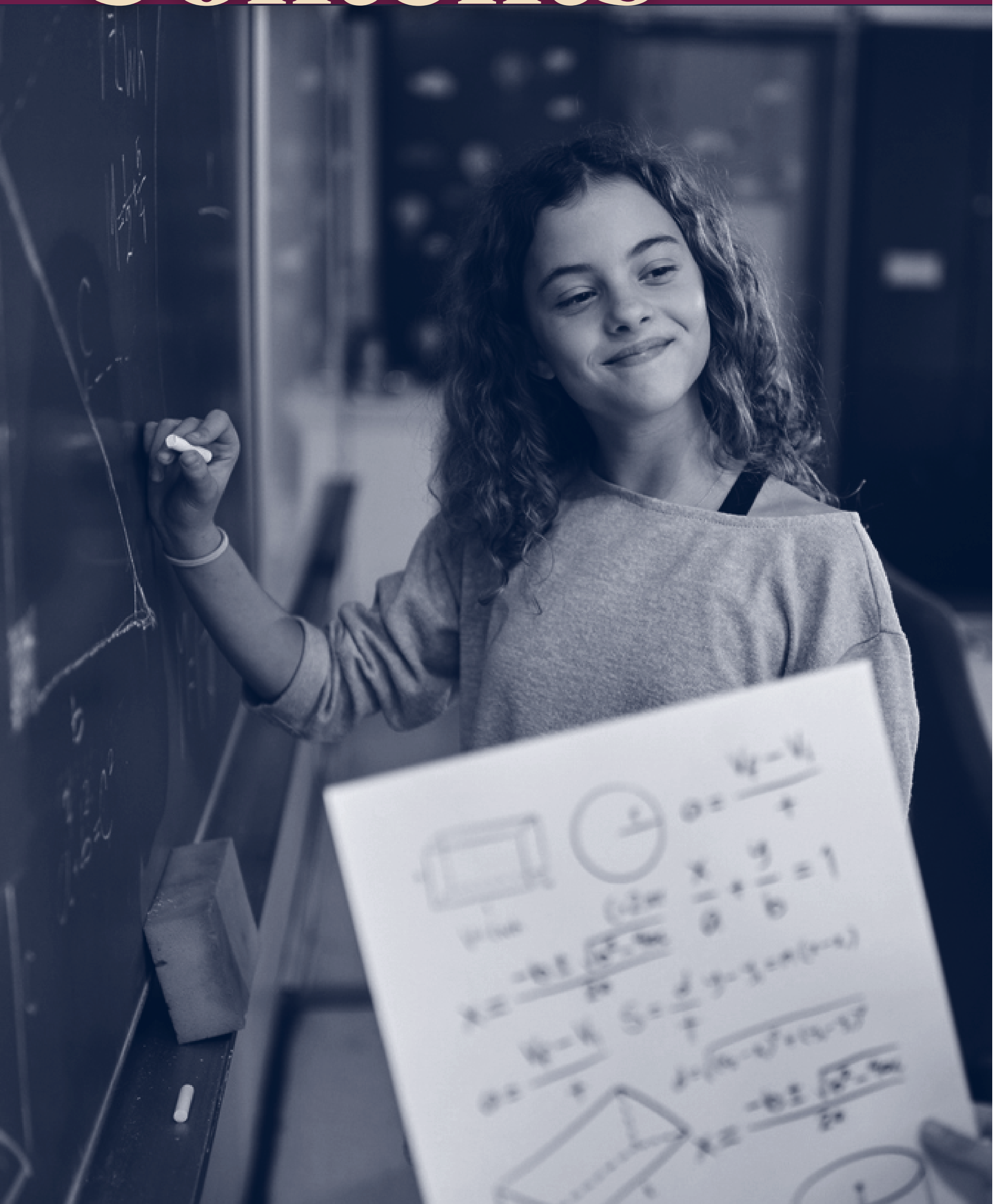

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Beta by Bellwether is an initiative that jump-starts action by designing big solutions to education's toughest challenges. Beta accelerates the development of new ideas, delivers them to the field, and models a new way forward for the sector — creating blueprints and tools that help leaders across the country build an education system that better serves all young people, particularly those furthest from opportunity. For more, visit **bellwether.org/beta**.

ACKNOWLEDGMENTS

We would like to thank the many experts who gave their time and shared their knowledge with us to inform our work, including Sarah Powell, Liz Cohen, and Joel Rose. Thank you also to the Valhalla Foundation and the Walton Family Foundation for their financial support of this project.

We would also like to thank our Bellwether colleagues Emily Shisler, Nate Geller, Michelle Croft, Bonnie O'Keefe, Titilayo Tinubu Ali, and Kelly Robson Foster for their input, Alexis Richardson for her support, and former colleague Sharmila Mann for her early and thorough research and insights. Thank you to Amy Ribock, Kate Stein, Andy Jacob, McKenzie Maxson, Temim Fruchter, Julie Nguyen, and Amber Walker for shepherding and disseminating this work, and to LJ Carter Creative and Super Copy Editors.

The contributions of these individuals and entities significantly enhanced our work; however, any errors in fact or analysis remain the responsibility of the authors.

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